GENERAL RULES OF INTEGRATION

In the following, u, v, w are functions of x; a, b, p, q, n any cons $e = 2.71828\ldots$ is the natural base of logarithms; $\ln u$ denotes the natural log: that $u > 0$ [in general, to extend formulas to cases where $u < 0$ as well, r are in radians; all constants of integration are omitted but implied.

14.1 $\int a\,dx = ax$

14.2 $\int af(x)\,dx = a\int f(x)\,dx$

14.3 $\int (u \pm v \pm w \pm \cdots)\,dx = \int u\,dx \pm \int v\,dx \pm \int w\,dx \pm \cdots$

14.4 $\int u\,dv = uv - \int v\,du$ [Integration by parts]

For generalized integration by parts, see 14.48.

14.5 $\int f(ax)\,dx = \frac{1}{a}\int f(u)\,du$

14.6 $\int F\{f(x)\}\,dx = \int F(u)\frac{dx}{du}\,du = \int \frac{F(u)}{f'(x)}\,du$ where $u = f(x)$

14.7 $\int u^n\,du = \frac{u^{n+1}}{n+1}, \quad n \neq -1$ [For $n = -1$, see 14.8]

14.8 $\int \frac{du}{u} = \ln u$ if $u > 0$ or $\ln(-u)$ if $u < 0$
$= \ln|u|$

14.9 $\int e^u\,du = e^u$

14.10 $\int a^u\,du = \int e^{u\ln a}\,du = \frac{e^{u\ln a}}{\ln a} = \frac{a^u}{\ln a}, \quad a > 0,\ a \neq 1$

14.11 $\int \sin u\,du = -\cos u$

14.12 $\int \cos u\,du = \sin u$

14.13 $\int \tan u\,du = \ln\sec u = -\ln\cos u$

14.14 $\int \cot u\,du = \ln\sin u$

14.15 $\int \sec u\,du = \ln(\sec u + \tan u) = \ln\tan\left(\frac{u}{2} + \frac{\pi}{4}\right)$

14.16 $\int \csc u\,du = \ln(\csc u - \cot u) = \ln\tan\frac{u}{2}$

14.17 $\int \sec^2 u\,du = \tan u$

14.18 $\int \csc^2 u\,du = -\cot u$

14.19 $\int \tan^2 u\,du = \tan u - u$

14.20 $\int \cot^2 u\,du = -\cot u - u$

14.21 $\int \sin^2 u\,du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u\cos u)$

14.22 $\int \cos^2 u\,du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u\cos u)$

14.23 $\int \sec u\tan u\,du = \sec u$

14.24 $\int \csc u\cot u\,du = -\csc u$

14.25 $\int \sinh u\,du = \cosh u$

14.26 $\int \cosh u\,du = \sinh u$

14.27 $\int \tanh u\,du = \ln\cosh u$

14.28 $\int \coth u\,du = \ln\sinh u$

14.29 $\int \operatorname{sech} u\,du = \sin^{-1}(\tanh u)$ or $2\tan^{-1}e^u$

14.30 $\int \operatorname{csch} u\,du = \ln\tanh\frac{u}{2}$ or $-\coth^{-1}e^u$

14.31 $\int \operatorname{sech}^2 u\,du = \tanh u$

14.32 $\int \operatorname{csch}^2 u\,du = -\coth u$

14.33 $\int \tanh^2 u\,du = u - \tanh u$

14.34 $\int \coth^2 u\,du = u - \coth u$

14.35 $\int \sinh^2 u\,du = \frac{\sinh 2u}{4} - \frac{u}{2} = \frac{1}{2}(\sinh u\cosh u - u)$

14.36 $\int \cosh^2 u\,du = \frac{\sinh 2u}{4} + \frac{u}{2} = \frac{1}{2}(\sinh u\cosh u + u)$

14.37 $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u$

14.38 $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u$

14.39 $\int \frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$

14.40 $\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right) = -\frac{1}{a}\coth^{-1}\frac{u}{a} \quad u^2 > a^2$

14.41 $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left(\frac{a+u}{a-u}\right) = \frac{1}{a}\tanh^{-1}\frac{u}{a} \quad u^2 < a^2$

14.42 $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a}$

14.43 $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$ or $\sinh^{-1}\frac{u}{a}$

14.44 $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$

14.45 $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right|$

14.46 $\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right)$

14.47 $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right)$

14.48 $\int f^{(n)}g\,dx = f^{(n-1)}g - f^{(n-2)}g' + f^{(n-3)}g'' - \cdots (-1)^n\int fg^{(n)}\,dx$

This is called *generalized integration* by parts.

IMPORTANT TRANSFORMATIONS

Often in practice an integral can be simplified by using an appropriate transfor and formula 14.6, page 57. The following list gives some transformations and their

14.49 $\int F(ax+b)\,dx = \frac{1}{a}\int F(u)\,du$ where $u = ax + b$

14.50 $\int F(\sqrt{ax+b})\,dx = \frac{2}{a}\int uF(u)\,du$ where $u = \sqrt{ax+b}$

14.51 $\int F(\sqrt[n]{ax+b})\,dx = \frac{n}{a}\int u^{n-1}F(u)\,du$ where $u = \sqrt[n]{ax+b}$

14.52 $\int F(\sqrt{a^2 - x^2})\,dx = a\int F(a\cos u)\cos u\,du$ where $x = a\sin u$

14.53 $\int F(\sqrt{x^2 + a^2})\,dx = a\int F(a\sec u)\sec^2 u\,du$ where $x = a\tan u$

14.54 $\int F(\sqrt{x^2 - a^2})\,dx = a\int F(a\tan u)\sec u\tan u\,du$ where $x = a\sec u$

14.55 $\int F(e^{ax})\,dx = \frac{1}{a}\int \frac{F(u)}{u}\,du$ where $u = e^{ax}$

14.56 $\int F(\ln x)\,dx = \int F(u)e^u\,du$ where $u = \ln x$

14.57 $\int F\left(\sin^{-1}\frac{x}{a}\right)dx = a\int F(u)\cos u\,du$ where $u = \sin^{-1}\frac{x}{a}$

Similar results apply for other inverse trigonometric functions.

14.58 $\int F(\sin x, \cos x)\,dx = 2\int F\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right)\frac{du}{1+u^2}$ where $u = \tan\frac{x}{2}$

INTEGRALS INVOLVING $ax + b$

14.59 $$\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$$

14.60 $$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$$

14.61 $$\int \frac{x^2\,dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$$

14.62 $$\int \frac{x^3\,dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b)$$

14.63 $$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$$

14.64 $$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$$

14.65 $$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right)$$

14.66 $$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

14.67 $$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$$

14.68 $$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b)$$

14.69 $$\int \frac{x^3\,dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{3b^2}{a^4}\ln(ax+b)$$

14.70 $$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$$

14.71 $$\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$$

14.72 $$\int \frac{dx}{x^3(ax+b)^2} = -\frac{(ax+b)^2}{2b^4x^2} + \frac{3a(ax+b)}{b^4x} - \frac{a^3x}{b^4(ax+b)} - \frac{3a^2}{b^4}\ln\left(\frac{ax+b}{x}\right)$$

14.73 $$\int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$$

14.74 $$\int \frac{x\,dx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$$

14.75 $$\int \frac{x^2\,dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3}\ln(ax+b)$$

14.76 $$\int \frac{x^3\,dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)^2} - \frac{3b}{a^4}\ln(ax+b)$$

14.77 $$\int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3}\ln\left(\frac{ax+b}{x}\right)$$

14.78 $$\int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4}\ln\left(\frac{ax+b}{x}\right)$$

14.79 $$\int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^2}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5}\ln\left(\frac{ax+b}{x}\right)$$

14.80 $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a}. \quad \text{If } n=-1, \text{ see 14.59.}$$

14.81 $$\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}, \quad n \neq -1, -2$$

If $n = -1, -2$, see 14.60, 14.67.

14.82 $$\int x^2(ax+b)^n\,dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{2b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}$$

If $n = -1, -2, -3$, see 14.61, 14.68, 14.75.

14.83 $$\int x^m(ax+b)^n\,dx = \begin{cases} \dfrac{x^{m+1}(ax+b)^n}{m+n+1} + \dfrac{nb}{m+n+1}\displaystyle\int x^m(ax+b)^{n-1}\,dx \\ \dfrac{x^m(ax+b)^{n+1}}{(m+n+1)a} - \dfrac{mb}{(m+n+1)a}\displaystyle\int x^{m-1}(ax+b)^n\,dx \\ \dfrac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \dfrac{m+n+2}{(n+1)b}\displaystyle\int x^m(ax+b)^{n+1}\,dx \end{cases}$$

INTEGRALS INVOLVING $\sqrt{ax+b}$

14.84 $$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

14.85 $$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

14.86 $$\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b}$$

14.87 $$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{b}}\ln\left(\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \dfrac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\dfrac{ax+b}{-b}} \end{cases}$$

14.88 $$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} \quad \text{[See 14.87]}$$

14.89 $$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

14.90 $$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$$

14.91 $$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3}$$

14.92 $$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}} \quad \text{[See 14.87]}$$

14.93 $$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}} \quad \text{[See 14.87]}$$

14.94 $$\int \frac{x^m}{\sqrt{ax+b}}\,dx = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int \frac{x^{m-1}}{\sqrt{ax+b}}\,dx$$

14.95 $$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$$

14.96 $$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}(ax+b)^{3/2} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx$$

14.97 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$$

14.98 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = \frac{-(ax+b)^{3/2}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int \frac{\sqrt{ax+b}}{x^{m-1}}\,dx$$

14.99 $$\int (ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+2)/2}}{a(m+2)}$$

14.100 $$\int x(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(ax+b)^{(m+2)/2}}{a^2(m+2)}$$

14.101 $$\int x^2(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+6)/2}}{a^3(m+6)} - \frac{4b(ax+b)^{(m+4)/2}}{a^3(m+4)} + \frac{2b^2(ax+b)^{(m+2)/2}}{a^3(m+2)}$$

14.102 $$\int \frac{(ax+b)^{m/2}}{x}\,dx = \frac{2(ax+b)^{m/2}}{m} + b\int \frac{(ax+b)^{(m-2)/2}}{x}\,dx$$

14.103 $$\int \frac{(ax+b)^{m/2}}{x^2}\,dx = -\frac{(ax+b)^{(m+2)/2}}{bx} + \frac{ma}{2b}\int \frac{(ax+b)^{m/2}}{x}\,dx$$

14.104 $$\int \frac{dx}{x(ax+b)^{m/2}} = \frac{2}{(m-2)b(ax+b)^{(m-2)/2}} + \frac{1}{b}\int \frac{dx}{x(ax+b)^{(m-2)/2}}$$

INTEGRALS INVOLVING $ax + b$ AND $px + q$

14.105 $$\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)$$

14.106 $$\int \frac{x\,dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\left\{\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\right\}$$

14.107 $$\int \frac{dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)\right\}$$

14.108 $$\int \frac{x\,dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{q}{bp-aq}\ln\left(\frac{ax+b}{px+q}\right) - \frac{b}{a(ax+b)}\right\}$$

14.110 $$\int \frac{dx}{(ax+b)^m(px+q)^n} = \frac{-1}{(n-1)(bp-aq)}\left\{\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int \frac{dx}{(ax+b)^m(px+q)^{n-1}}\right\}$$

14.111 $$\int \frac{ax+b}{px+q}\,dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$$

14.112 $$\int \frac{(ax+b)^m}{(px+q)^n}\,dx = \begin{cases} \dfrac{-1}{(n-1)(bp-aq)}\left\{\dfrac{(ax+b)^{m+1}}{(px+q)^{n-1}} + (n-m-2)a\displaystyle\int \frac{(ax+b)^m}{(px+q)^{n-1}}\,dx\right\} \\ \dfrac{-1}{(n-m-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\displaystyle\int \frac{(ax+b)^{m-1}}{(px+q)^n}\,dx\right\} \\ \dfrac{-1}{(n-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} - ma\displaystyle\int \frac{(ax+b)^{m-1}}{(px+q)^{n-1}}\,dx\right\} \end{cases}$$

14.109 $$\int \frac{x^2\,dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2}\left\{\frac{q^2}{p}\ln(px+q) + \frac{b(bp-2aq)}{a^2}\ln(ax+b)\right\}$$

INTEGRALS INVOLVING $\sqrt{ax+b}$ AND $px+q$

14.113 $$\int \frac{px+q}{\sqrt{ax+b}}\,dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$$

14.114 $$\int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{bp-aq}\,\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2}{\sqrt{aq-bp}\,\sqrt{p}}\tan^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

14.115 $$\int \frac{\sqrt{ax+b}}{px+q}dx = \begin{cases} \dfrac{2\sqrt{ax+b}}{p} + \dfrac{\sqrt{bp-aq}}{p\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2\sqrt{ax+b}}{p} - \dfrac{2\sqrt{aq-bp}}{p\sqrt{p}}\tan^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

14.116 $$\int (px+q)^n\sqrt{ax+b}\,dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p}\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx$$

14.117 $$\int \frac{dx}{(px+q)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)(aq-bp)}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

14.118 $$\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx = \frac{2(px+q)^n\sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n+1)a}\int \frac{(px+q)^{n-1}\,dx}{\sqrt{ax+b}}$$

14.119 $$\int \frac{\sqrt{ax+b}}{(px+q)^n}\,dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

INTEGRALS INVOLVING $\sqrt{ax+b}$ AND $\sqrt{px+q}$

14.120 $$\int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases} \dfrac{2}{\sqrt{ap}}\ln\left(\sqrt{a(px+q)}+\sqrt{p(ax+b)}\right) \\ \dfrac{2}{\sqrt{-ap}}\tan^{-1}\sqrt{\dfrac{-p(ax+b)}{a(px+q)}} \end{cases}$$

14.121 $$\int \frac{x\,dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

14.122 $$\int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

14.123 $$\int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{aq-bp}{2a}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

14.124 $$\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

INTEGRALS INVOLVING x^2+a^2

14.125 $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

14.126 $$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2)$$

14.127 $$\int \frac{x^2\,dx}{x^2+a^2} = x - a\tan^{-1}\frac{x}{a}$$

14.128 $$\int \frac{x^3\,dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2}\ln(x^2+a^2)$$

14.129 $$\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

14.130 $$\int \frac{dx}{x^2(x^2+a^2)} = -\frac{1}{a^2x} - \frac{1}{a^3}\tan^{-1}\frac{x}{a}$$

14.131 $$\int \frac{dx}{x^3(x^2+a^2)} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

14.132 $$\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\tan^{-1}\frac{x}{a}$$

14.133 $$\int \frac{x\,dx}{(x^2+a^2)^2} = \frac{-1}{2(x^2+a^2)}$$

14.134 $$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a}\tan^{-1}\frac{x}{a}$$

14.135 $$\int \frac{x^3\,dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2}\ln(x^2+a^2)$$

14.136 $$\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

14.137 $$\int \frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5}\tan^{-1}\frac{x}{a}$$

14.138 $$\int \frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^6}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

14.139 $$\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}$$

14.140 $$\int \frac{x\,dx}{(x^2+a^2)^n} = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$$

14.141 $$\int \frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(x^2+a^2)^{n-1}}$$

14.142 $$\int \frac{x^m\,dx}{(x^2+a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2+a^2)^{n-1}} - a^2\int \frac{x^{m-2}\,dx}{(x^2+a^2)^n}$$

14.143 $$\int \frac{dx}{x^m(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2+a^2)^n}$$

INTEGRALS INVOLVING $x^2-a^2, \quad x^2>a^2$

14.144 $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) \quad \text{or} \quad -\frac{1}{a}\coth^{-1}\frac{x}{a}$$

14.145 $$\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln(x^2-a^2)$$

14.146 $$\int \frac{x^2\,dx}{x^2-a^2} = x + \frac{a}{2}\ln\left(\frac{x-a}{x+a}\right)$$

14.147 $$\int \frac{x^3\,dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2}\ln(x^2-a^2)$$

14.148 $$\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2-a^2}{x^2}\right)$$

14.149 $$\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{x-a}{x+a}\right)$$

14.150 $$\int \frac{dx}{x^3(x^2-a^2)} = \frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$$

14.151 $$\int \frac{dx}{(x^2-a^2)^2} = \frac{-x}{2a^2(x^2-a^2)} - \frac{1}{4a^3}\ln\left(\frac{x-a}{x+a}\right)$$

14.152 $$\int \frac{x\,dx}{(x^2-a^2)^2} = \frac{-1}{2(x^2-a^2)}$$

14.153 $$\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{-x}{2(x^2-a^2)} + \frac{1}{4a}\ln\left(\frac{x-a}{x+a}\right)$$

14.154 $$\int \frac{x^3\,dx}{(x^2-a^2)^2} = \frac{-a^2}{2(x^2-a^2)} + \frac{1}{2}\ln(x^2-a^2)$$

14.155 $$\int \frac{dx}{x(x^2-a^2)^2} = \frac{-1}{2a^2(x^2-a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$$

14.156 $$\int \frac{dx}{x^2(x^2-a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2-a^2)} - \frac{3}{4a^5}\ln\left(\frac{x-a}{x+a}\right)$$

14.157 $$\int \frac{dx}{x^3(x^2-a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2-a^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{x^2-a^2}\right)$$

14.158 $$\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2-a^2)^{n-1}}$$

14.159 $$\int \frac{x\,dx}{(x^2-a^2)^n} = \frac{-1}{2(n-1)(x^2-a^2)^{n-1}}$$

14.160 $$\int \frac{dx}{x(x^2-a^2)^n} = \frac{-1}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x(x^2-a^2)^{n-1}}$$

14.161 $$\int \frac{x^m\,dx}{(x^2-a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2-a^2)^{n-1}} + a^2\int \frac{x^{m-2}\,dx}{(x^2-a^2)^n}$$

14.162 $$\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2-a^2)^n} - \frac{1}{a^2}\int \frac{dx}{x^m(x^2-a^2)^{n-1}}$$

INTEGRALS INVOLVING $a^2-x^2, \quad x^2<a^2$

14.163 $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) \quad \text{or} \quad \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

14.164 $$\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln(a^2-x^2)$$

14.165 $$\int \frac{x^2\,dx}{a^2-x^2} = -x + \frac{a}{2}\ln\left(\frac{a+x}{a-x}\right)$$

14.166 $$\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$$

14.167 $$\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

14.168 $$\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{a+x}{a-x}\right)$$

14.169 $$\int \frac{dx}{x^3(a^2-x^2)} = -\frac{1}{2a^2x^2} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

14.170 $$\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3}\ln\left(\frac{a+x}{a-x}\right)$$

14.171 $$\int \frac{x\,dx}{(a^2-x^2)^2} = \frac{1}{2(a^2-x^2)}$$

14.172 $$\int \frac{x^2\,dx}{(a^2-x^2)^2} = \frac{x}{2(a^2-x^2)} - \frac{1}{4a}\ln\left(\frac{a+x}{a-x}\right)$$

14.173 $$\int \frac{x^3\,dx}{(a^2-x^2)^2} = \frac{a^2}{2(a^2-x^2)} + \frac{1}{2}\ln(a^2-x^2)$$

14.174 $$\int \frac{dx}{x(a^2-x^2)^2} = \frac{1}{2a^2(a^2-x^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

14.175 $$\int \frac{dx}{x^2(a^2-x^2)^2} = \frac{-1}{a^4x} + \frac{x}{2a^4(a^2-x^2)} + \frac{3}{4a^5}\ln\left(\frac{a+x}{a-x}\right)$$

14.176 $$\int \frac{dx}{x^3(a^2-x^2)^2} = \frac{-1}{2a^4x^2} + \frac{1}{2a^4(a^2-x^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

14.177 $$\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(a^2-x^2)^{n-1}}$$

14.178 $$\int \frac{x\,dx}{(a^2-x^2)^n} = \frac{1}{2(n-1)(a^2-x^2)^{n-1}}$$

14.179 $$\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2-x^2)^{n-1}}$$

14.180 $$\int \frac{x^m\,dx}{(a^2-x^2)^n} = a^2\int \frac{x^{m-2}dx}{(a^2-x^2)^n} - \int \frac{x^{m-2}dx}{(a^2-x^2)^{n-1}}$$

14.181 $$\int \frac{dx}{x^m(a^2-x^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x^{m-2}(a^2-x^2)^n}$$

INTEGRALS INVOLVING $\sqrt{x^2+a^2}$

14.182 $$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) \quad \text{or} \quad \sinh^{-1}\frac{x}{a}$$

14.183 $$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

14.184 $$\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.185 $$\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{(x^2+a^2)^{3/2}}{3} - a^2\sqrt{x^2+a^2}$$

14.186 $$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.187 $$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

14.188 $$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.189 $$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.190 $$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{3/2}}{3}$$

14.191 $$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.192 $$\int x^3\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{5/2}}{5} - \frac{a^2(x^2+a^2)^{3/2}}{3}$$

14.193 $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.194 $$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

14.195 $$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.196 $$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

14.197 $$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

14.198 $$\int \frac{x^2\,dx}{(x^2+a^2)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

14.199 $$\int \frac{x^3\,dx}{(x^2+a^2)^{3/2}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

14.200 $$\int \frac{dx}{x(x^2+a^2)^{3/2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.201 $$\int \frac{dx}{x^2(x^2+a^2)^{3/2}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

14.202 $$\int \frac{dx}{x^3(x^2+a^2)^{3/2}} = \frac{-1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.203 $$\int (x^2+a^2)^{3/2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3}{8}a^4\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.204 $$\int x(x^2+a^2)^{3/2}\,dx = \frac{(x^2+a^2)^{5/2}}{5}$$

14.205 $$\int x^2(x^2+a^2)^{3/2}\,dx = \frac{x(x^2+a^2)^{5/2}}{6} - \frac{a^2x(x^2+a^2)^{3/2}}{24} - \frac{a^4x\sqrt{x^2+a^2}}{16} - \frac{a^6}{16}\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.206 $$\int x^3(x^2+a^2)^{3/2}\,dx = \frac{(x^2+a^2)^{7/2}}{7} - \frac{a^2(x^2+a^2)^{5/2}}{5}$$

14.207 $$\int \frac{(x^2+a^2)^{3/2}}{x}\,dx = \frac{(x^2+a^2)^{3/2}}{3} + a^2\sqrt{x^2+a^2} - a^3\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.208 $$\int \frac{(x^2+a^2)^{3/2}}{x^2}\,dx = -\frac{(x^2+a^2)^{3/2}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2}a^2\ln\left(x+\sqrt{x^2+a^2}\right)$$

14.209 $$\int \frac{(x^2+a^2)^{3/2}}{x^3}\,dx = -\frac{(x^2+a^2)^{3/2}}{2x^2} + \frac{3}{2}\sqrt{x^2+a^2} - \frac{3}{2}a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

INTEGRALS INVOLVING $\sqrt{x^2-a^2}$

14.210 $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right), \qquad \int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

14.211 $$\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.212 $$\int \frac{x^3\,dx}{\sqrt{x^2-a^2}} = \frac{(x^2-a^2)^{3/2}}{3} + a^2\sqrt{x^2-a^2}$$

14.213 $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$$

14.214 $$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$$

14.215 $$\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

14.216 $$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.217 $$\int x\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{3/2}}{3}$$

14.218 $$\int x^2\sqrt{x^2-a^2}\,dx = \frac{x(x^2-a^2)^{3/2}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.219 $$\int x^3\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{5/2}}{5} + \frac{a^2(x^2-a^2)^{3/2}}{3}$$

14.220 $$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$$

14.221 $$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left(x+\sqrt{x^2-a^2}\right)$$

14.222 $$\int \frac{\sqrt{x^2-a^2}}{x^3}\,dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\sec^{-1}\left|\frac{x}{a}\right|$$

14.223 $$\int \frac{dx}{(x^2-a^2)^{3/2}} = -\frac{x}{a^2\sqrt{x^2-a^2}}$$

14.224 $$\int \frac{x\,dx}{(x^2-a^2)^{3/2}} = \frac{-1}{\sqrt{x^2-a^2}}$$

14.225 $$\int \frac{x^2\,dx}{(x^2-a^2)^{3/2}} = -\frac{x}{\sqrt{x^2-a^2}} + \ln\left(x+\sqrt{x^2-a^2}\right)$$

14.226 $$\int \frac{x^3\,dx}{(x^2-a^2)^{3/2}} = \sqrt{x^2-a^2} - \frac{a^2}{\sqrt{x^2-a^2}}$$

14.227 $$\int \frac{dx}{x(x^2-a^2)^{3/2}} = \frac{-1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

14.228 $$\int \frac{dx}{x^2(x^2-a^2)^{3/2}} = -\frac{\sqrt{x^2-a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2-a^2}}$$

14.229 $$\int \frac{dx}{x^3(x^2-a^2)^{3/2}} = \frac{1}{2a^2x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5}\sec^{-1}\left|\frac{x}{a}\right|$$

14.230 $$\int (x^2-a^2)^{3/2}\,dx = \frac{x(x^2-a^2)^{3/2}}{4} - \frac{3a^2x\sqrt{x^2-a^2}}{8} + \frac{3}{8}a^4\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.231 $$\int x(x^2-a^2)^{3/2}\,dx = \frac{(x^2-a^2)^{5/2}}{5}$$

14.232 $$\int x^2(x^2-a^2)^{3/2}\,dx = \frac{x(x^2-a^2)^{5/2}}{6} + \frac{a^2x(x^2-a^2)^{3/2}}{24} - \frac{a^4x\sqrt{x^2-a^2}}{16} + \frac{a^6}{16}\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.233 $$\int x^3(x^2-a^2)^{3/2}\,dx = \frac{(x^2-a^2)^{7/2}}{7} + \frac{a^2(x^2-a^2)^{5/2}}{5}$$

14.234 $$\int \frac{(x^2-a^2)^{3/2}}{x}\,dx = \frac{(x^2-a^2)^{3/2}}{3} - a^2\sqrt{x^2-a^2} + a^3\sec^{-1}\left|\frac{x}{a}\right|$$

14.235 $$\int \frac{(x^2-a^2)^{3/2}}{x^2}\,dx = -\frac{(x^2-a^2)^{3/2}}{x} + \frac{3x\sqrt{x^2-a^2}}{2} - \frac{3}{2}a^2\ln\left(x+\sqrt{x^2-a^2}\right)$$

14.236 $$\int \frac{(x^2-a^2)^{3/2}}{x^3}\,dx = -\frac{(x^2-a^2)^{3/2}}{2x^2} + \frac{3\sqrt{x^2-a^2}}{2} - \frac{3}{2}a\sec^{-1}\left|\frac{x}{a}\right|$$

INTEGRALS INVOLVING $\sqrt{a^2-x^2}$

14.237 $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

14.238 $$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

14.239 $$\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

14.240 $$\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \frac{(a^2-x^2)^{3/2}}{3} - a^2\sqrt{a^2-x^2}$$

14.241 $$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.242 $$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$$

14.243 $$\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} - \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.244 $$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

14.245 $$\int x\sqrt{a^2-x^2}\,dx = -\frac{(a^2-x^2)^{3/2}}{3}$$

14.246 $$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x(a^2-x^2)^{3/2}}{4} + \frac{a^2x\sqrt{a^2-x^2}}{8} + \frac{a^4}{8}\sin^{-1}\frac{x}{a}$$

14.247 $$\int x^3\sqrt{a^2-x^2}\,dx = \frac{(a^2-x^2)^{5/2}}{5} - \frac{a^2(a^2-x^2)^{3/2}}{3}$$

14.248 $$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.249 $$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \sin^{-1}\frac{x}{a}$$

14.250 $$\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{a^2-x^2}}{2x^2} + \frac{1}{2a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.251 $$\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

14.252 $$\int \frac{x\,dx}{(a^2-x^2)^{3/2}} = \frac{1}{\sqrt{a^2-x^2}}$$

14.253 $$\int \frac{x^2\,dx}{(a^2-x^2)^{3/2}} = \frac{x}{\sqrt{a^2-x^2}} - \sin^{-1}\frac{x}{a}$$

14.254 $$\int \frac{x^3\,dx}{(a^2-x^2)^{3/2}} = \sqrt{a^2-x^2} + \frac{a^2}{\sqrt{a^2-x^2}}$$

14.255 $$\int \frac{dx}{x(a^2-x^2)^{3/2}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.256 $$\int \frac{dx}{x^2(a^2-x^2)^{3/2}} = -\frac{\sqrt{a^2-x^2}}{a^4x} + \frac{x}{a^4\sqrt{a^2-x^2}}$$

14.257 $$\int \frac{dx}{x^3(a^2-x^2)^{3/2}} = \frac{-1}{2a^2x^2\sqrt{a^2-x^2}} + \frac{3}{2a^4\sqrt{a^2-x^2}} - \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.258 $$\int (a^2-x^2)^{3/2}\,dx = \frac{x(a^2-x^2)^{3/2}}{4} + \frac{3a^2x\sqrt{a^2-x^2}}{8} + \frac{3}{8}a^4\sin^{-1}\frac{x}{a}$$

14.259 $$\int x(a^2-x^2)^{3/2}\,dx = -\frac{(a^2-x^2)^{5/2}}{5}$$

14.260 $$\int x^2(a^2-x^2)^{3/2}\,dx = -\frac{x(a^2-x^2)^{5/2}}{6} + \frac{a^2x(a^2-x^2)^{3/2}}{24} + \frac{a^4x\sqrt{a^2-x^2}}{16} + \frac{a^6}{16}\sin^{-1}\frac{x}{a}$$

14.261 $$\int x^3(a^2-x^2)^{3/2}\,dx = \frac{(a^2-x^2)^{7/2}}{7} - \frac{a^2(a^2-x^2)^{5/2}}{5}$$

14.262 $$\int \frac{(a^2-x^2)^{3/2}}{x}\,dx = \frac{(a^2-x^2)^{3/2}}{3} + a^2\sqrt{a^2-x^2} - a^3\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

14.263 $$\int \frac{(a^2-x^2)^{3/2}}{x^2}\,dx = -\frac{(a^2-x^2)^{3/2}}{x} - \frac{3x\sqrt{a^2-x^2}}{2} - \frac{3}{2}a^2\sin^{-1}\frac{x}{a}$$

14.264 $$\int \frac{(a^2-x^2)^{3/2}}{x^3}\,dx = -\frac{(a^2-x^2)^{3/2}}{2x^2} - \frac{3\sqrt{a^2-x^2}}{2} + \frac{3}{2}a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

INTEGRALS INVOLVING ax^2+bx+c

14.265 $$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\tan^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}} \\ \dfrac{1}{\sqrt{b^2-4ac}}\ln\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right) \end{cases}$$

If $b^2=4ac$, $ax^2+bx+c=a(x+b/2a)^2$ and the results on pages 60-61 can be used. If a or $c=0$ use results on page 64. If a or $c=0$ use results on pages 60-61.

14.266 $$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c}$$

14.267 $$\int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2}\int \frac{dx}{ax^2+bx+c}$$

14.268 $$\int \frac{x^m\,dx}{ax^2+bx+c} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a}\int \frac{x^{m-2}\,dx}{ax^2+bx+c} - \frac{b}{a}\int \frac{x^{m-1}\,dx}{ax^2+bx+c}$$

14.269 $$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left(\frac{x^2}{ax^2+bx+c}\right) - \frac{b}{2c}\int \frac{dx}{ax^2+bx+c}$$

14.270 $$\int \frac{dx}{x^2(ax^2+bx+c)} = \frac{b}{2c^2}\ln\left(\frac{ax^2+bx+c}{x^2}\right) - \frac{1}{cx} + \frac{b^2-2ac}{2c^2}\int \frac{dx}{ax^2+bx+c}$$

14.271 $$\int \frac{dx}{x^n(ax^2+bx+c)} = -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c}\int \frac{dx}{x^{n-1}(ax^2+bx+c)} - \frac{a}{c}\int \frac{dx}{x^{n-2}(ax^2+bx+c)}$$

14.272 $$\int \frac{dx}{(ax^2+bx+c)^2} = \frac{2ax+b}{(4ac-b^2)(ax^2+bx+c)} + \frac{2a}{4ac-b^2}\int \frac{dx}{ax^2+bx+c}$$

14.273 $$\int \frac{x\,dx}{(ax^2+bx+c)^2} = -\frac{bx+2c}{(4ac-b^2)(ax^2+bx+c)} - \frac{b}{4ac-b^2}\int \frac{dx}{ax^2+bx+c}$$

14.274 $$\int \frac{x^2\,dx}{(ax^2+bx+c)^2} = \frac{(b^2-2ac)x+bc}{a(4ac-b^2)(ax^2+bx+c)} + \frac{2c}{4ac-b^2}\int \frac{dx}{ax^2+bx+c}$$

14.275 $$\int \frac{x^m\,dx}{(ax^2+bx+c)^n} = -\frac{x^{m-1}}{(2n-m-1)a(ax^2+bx+c)^{n-1}} + \frac{(m-1)c}{(2n-m-1)a}\int \frac{x^{m-2}\,dx}{(ax^2+bx+c)^n} - \frac{(n-m)b}{(2n-m-1)a}\int \frac{x^{m-1}\,dx}{(ax^2+bx+c)^n}$$

14.276 $$\int \frac{x^{2n-1}\,dx}{(ax^2+bx+c)^n} = \frac{1}{a}\int \frac{x^{2n-3}\,dx}{(ax^2+bx+c)^{n-1}} - \frac{c}{a}\int \frac{x^{2n-3}\,dx}{(ax^2+bx+c)^n} - \frac{b}{a}\int \frac{x^{2n-2}\,dx}{(ax^2+bx+c)^n}$$

14.277 $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{1}{2c(ax^2+bx+c)} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^2} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)}$$

14.278 $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = -\frac{1}{cx(ax^2+bx+c)} - \frac{3a}{c}\int \frac{dx}{(ax^2+bx+c)^2} - \frac{2b}{c}\int \frac{dx}{x(ax^2+bx+c)^2}$$

14.279 $$\int \frac{dx}{x^m(ax^2+bx+c)^n} = -\frac{1}{(m-1)cx^{m-1}(ax^2+bx+c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c}\int \frac{dx}{x^{m-2}(ax^2+bx+c)^n} - \frac{(m+n-2)b}{(m-1)c}\int \frac{dx}{x^{m-1}(ax^2+bx+c)^n}$$

INTEGRALS INVOLVING $\sqrt{ax^2+bx+c}$

In the following results if $b^2=4ac$, $\sqrt{ax^2+bx+c}=\sqrt{a}\,(x+b/2a)$ and the results on pages
be used. If $b=0$ use the results on pages 67-70. If $a=0$ or $c=0$ use the results on pages

14.280 $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left(2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right) \\ -\dfrac{1}{\sqrt{-a}}\sin^{-1}\left(\dfrac{2ax+b}{\sqrt{b^2-4ac}}\right) \text{ or } \dfrac{1}{\sqrt{a}}\sinh^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$$

14.281 $$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

14.282 $$\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

14.283 $$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\dfrac{1}{\sqrt{c}}\ln\left(\dfrac{2\sqrt{c}\sqrt{ax^2+bx+c}+bx+2c}{x}\right) \\ \dfrac{1}{\sqrt{-c}}\sin^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) \text{ or } -\dfrac{1}{\sqrt{c}}\sinh^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$$

14.284 $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

14.285 $$\int \sqrt{ax^2+bx+c}\,dx = \frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a} + \frac{4ac-b^2}{8a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

14.286 $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{(ax^2+bx+c)^{3/2}}{3a} - \frac{b(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

14.287 $$\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{6ax-5b}{24a^2}(ax^2+bx+c)^{3/2} + \frac{5b^2-4ac}{16a^2}\int \sqrt{ax^2+bx+c}\,dx$$

14.288 $$\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int \frac{dx}{\sqrt{ax^2+bx+c}} + c\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

14.289 $$\int \frac{\sqrt{ax^2+bx+c}}{x^2}\,dx = -\frac{\sqrt{ax^2+bx+c}}{x} + a\int \frac{dx}{\sqrt{ax^2+bx+c}} + \frac{b}{2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

14.290 $$\int \frac{dx}{(ax^2+bx+c)^{3/2}} = \frac{2(2ax+b)}{(4ac-b^2)\sqrt{ax^2+bx+c}}$$

14.291 $$\int \frac{x\,dx}{(ax^2+bx+c)^{3/2}} = \frac{2(bx+2c)}{(b^2-4ac)\sqrt{ax^2+bx+c}}$$

14.292 $$\int \frac{x^2\,dx}{(ax^2+bx+c)^{3/2}} = \frac{(2b^2-4ac)x+2bc}{a(4ac-b^2)\sqrt{ax^2+bx+c}} + \frac{1}{a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

14.293 $$\int \frac{dx}{x(ax^2+bx+c)^{3/2}} = \frac{1}{c\sqrt{ax^2+bx+c}} + \frac{1}{c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^{3/2}}$$

14.294 $$\int \frac{dx}{x^2(ax^2+bx+c)^{3/2}} = -\frac{ax^2+2bx+c}{c^2x\sqrt{ax^2+bx+c}} + \frac{b^2-2ac}{2c^2}\int \frac{dx}{(ax^2+bx+c)^{3/2}} - \frac{3b}{2c^2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

14.295 $$\int (ax^2+bx+c)^{n+1/2}\,dx = \frac{(2ax+b)(ax^2+bx+c)^{n+1/2}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)}\int (ax^2+bx+c)^{n-1/2}\,dx$$

14.296 $$\int x(ax^2+bx+c)^{n+1/2}\,dx = \frac{(ax^2+bx+c)^{n+3/2}}{a(2n+3)} - \frac{b}{2a}\int (ax^2+bx+c)^{n+1/2}\,dx$$

14.297 $$\int \frac{dx}{(ax^2+bx+c)^{n+1/2}} = \frac{2(2ax+b)}{(2n-1)(4ac-b^2)(ax^2+bx+c)^{n-1/2}} + \frac{8a(n-1)}{(2n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1/2}}$$

14.298 $$\int \frac{dx}{x(ax^2+bx+c)^{n+1/2}} = \frac{1}{(2n-1)c(ax^2+bx+c)^{n-1/2}} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)^{n-1/2}} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^{n+1/2}}$$

INTEGRALS INVOLVING $x^3 + a^3$

Note that for formulas involving $x^3 - a^3$ replace a by $-a$.

14.299 $$\int \frac{dx}{x^3+a^3} = \frac{1}{6a^2}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{1}{a^2\sqrt{3}}\tan^{-1}\frac{2x-a}{a\sqrt{3}}$$

14.300 $$\int \frac{x\,dx}{x^3+a^3} = \frac{1}{6a}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{a\sqrt{3}}\tan^{-1}\frac{2x-a}{a\sqrt{3}}$$

14.301 $$\int \frac{x^2\,dx}{x^3+a^3} = \frac{1}{3}\ln(x^3+a^3)$$

14.302 $$\int \frac{dx}{x(x^3+a^3)} = \frac{1}{3a^3}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

14.303 $$\int \frac{dx}{x^2(x^3+a^3)} = -\frac{1}{a^3x} - \frac{1}{6a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} - \frac{1}{a^4\sqrt{3}}\tan^{-1}\frac{2x-a}{a\sqrt{3}}$$

14.304 $$\int \frac{dx}{(x^3+a^3)^2} = \frac{x}{3a^3(x^3+a^3)} + \frac{1}{9a^5}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{2}{3a^5\sqrt{3}}\tan^{-1}\frac{2x-a}{a\sqrt{3}}$$

14.305 $$\int \frac{x\,dx}{(x^3+a^3)^2} = \frac{x^2}{3a^3(x^3+a^3)} + \frac{1}{18a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{3a^4\sqrt{3}}\tan^{-1}\frac{2x-a}{a\sqrt{3}}$$

14.306 $$\int \frac{x^2\,dx}{(x^3+a^3)^2} = -\frac{1}{3(x^3+a^3)}$$

14.307 $$\int \frac{dx}{x(x^3+a^3)^2} = \frac{1}{3a^3(x^3+a^3)} + \frac{1}{3a^6}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

14.308 $$\int \frac{dx}{x^2(x^3+a^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(x^3+a^3)} - \frac{4}{3a^6}\int\frac{x\,dx}{x^3+a^3}$$ [See 14.300]

14.309 $$\int \frac{x^m\,dx}{x^3+a^3} = \frac{x^{m-2}}{m-2} - a^3\int\frac{x^{m-3}\,dx}{x^3+a^3}$$

14.310 $$\int \frac{dx}{x^n(x^3+a^3)} = \frac{-1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3}\int\frac{dx}{x^{n-3}(x^3+a^3)}$$

INTEGRALS INVOLVING $x^4 \pm a^4$

14.311 $$\int \frac{dx}{x^4+a^4} = \frac{1}{4a^3\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) - \frac{1}{2a^3\sqrt{2}}\tan^{-1}\frac{ax\sqrt{2}}{x^2-a^2}$$

14.312 $$\int \frac{x\,dx}{x^4+a^4} = \frac{1}{2a^2}\tan^{-1}\frac{x^2}{a^2}$$

14.313 $$\int \frac{x^2\,dx}{x^4+a^4} = \frac{1}{4a\sqrt{2}}\ln\left(\frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2}\right) - \frac{1}{2a\sqrt{2}}\tan^{-1}\frac{ax\sqrt{2}}{x^2-a^2}$$

14.314 $$\int \frac{x^3\,dx}{x^4+a^4} = \frac{1}{4}\ln(x^4+a^4)$$

14.315 $$\int \frac{dx}{x(x^4+a^4)} = \frac{1}{4a^4}\ln\left(\frac{x^4}{x^4+a^4}\right)$$

14.316 $$\int \frac{dx}{x^2(x^4+a^4)} = -\frac{1}{a^4x} - \frac{1}{4a^5\sqrt{2}}\ln\left(\frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2}\right) + \frac{1}{2a^5\sqrt{2}}\tan^{-1}\frac{ax\sqrt{2}}{x^2-a^2}$$

14.317 $$\int \frac{dx}{x^3(x^4+a^4)} = -\frac{1}{2a^4x^2} - \frac{1}{2a^6}\tan^{-1}\frac{x^2}{a^2}$$

14.318 $$\int \frac{dx}{x^4-a^4} = \frac{1}{4a^3}\ln\left(\frac{x-a}{x+a}\right) - \frac{1}{2a^3}\tan^{-1}\frac{x}{a}$$

14.319 $$\int \frac{x\,dx}{x^4-a^4} = \frac{1}{4a^2}\ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$$

14.320 $$\int \frac{x^2\,dx}{x^4-a^4} = \frac{1}{4a}\ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2a}\tan^{-1}\frac{x}{a}$$

14.321 $$\int \frac{x^3\,dx}{x^4-a^4} = \frac{1}{4}\ln(x^4-a^4)$$

14.322 $$\int \frac{dx}{x(x^4-a^4)} = \frac{1}{4a^4}\ln\left(\frac{x^4-a^4}{x^4}\right)$$

14.323 $$\int \frac{dx}{x^2(x^4-a^4)} = \frac{1}{a^4x} + \frac{1}{4a^5}\ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2a^5}\tan^{-1}\frac{x}{a}$$

14.324 $$\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6}\ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$$

INTEGRALS INVOLVING $x^n \pm a^n$

14.325 $$\int \frac{dx}{x(x^n+a^n)} = \frac{1}{na^n}\ln\frac{x^n}{x^n+a^n}$$

14.326 $$\int \frac{x^{n-1}\,dx}{x^n+a^n} = \frac{1}{n}\ln(x^n+a^n)$$

14.327 $$\int \frac{x^m\,dx}{(x^n+a^n)^r} = \int\frac{x^{m-n}\,dx}{(x^n+a^n)^{r-1}} - a^n\int\frac{x^{m-n}\,dx}{(x^n+a^n)^r}$$

14.328 $$\int \frac{dx}{x^m(x^n+a^n)^r} = \frac{1}{a^n}\int\frac{dx}{x^m(x^n+a^n)^{r-1}} - \frac{1}{a^n}\int\frac{dx}{x^{m-n}(x^n+a^n)}$$

14.329 $$\int \frac{dx}{x\sqrt{x^n+a^n}} = \frac{1}{n\sqrt{a^n}}\ln\left(\frac{\sqrt{x^n+a^n}-\sqrt{a^n}}{\sqrt{x^n+a^n}+\sqrt{a^n}}\right)$$

14.330 $$\int \frac{dx}{x(x^n-a^n)} = \frac{1}{na^n}\ln\left(\frac{x^n-a^n}{x^n}\right)$$

14.331 $$\int \frac{x^{n-1}\,dx}{x^n-a^n} = \frac{1}{n}\ln(x^n-a^n)$$

14.332 $$\int \frac{x^m\,dx}{(x^n-a^n)^r} = a^n\int\frac{x^{m-n}\,dx}{(x^n-a^n)^r} + \int\frac{x^{m-n}\,dx}{(x^n-a^n)^{r-1}}$$

14.333 $$\int \frac{dx}{x^m(x^n-a^n)^r} = \frac{1}{a^n}\int\frac{dx}{x^{m-n}(x^n-a^n)^r} - \frac{1}{a^n}\int\frac{dx}{x^m(x^n-a^n)^{r-1}}$$

14.334 $$\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}}\cos^{-1}\sqrt{\frac{a^n}{x^n}}$$

14.335 $$\int \frac{x^{p-1}\,dx}{x^{2m}+a^{2m}} = \frac{1}{ma^{2m-p}}\sum_{k=1}^{m}\sin\frac{(2k-1)p\pi}{2m}\tan^{-1}\left(\frac{x+a\cos[(2k-1)\pi/2m]}{a\sin[(2k-1)\pi/2m]}\right) - \frac{1}{2ma^{2m-p}}\sum_{k=1}^{m}\cos\frac{(2k-1)p\pi}{2m}\ln\left(x^2+2ax\cos\frac{(2k-1)\pi}{2m}+a^2\right)$$

where $0 < p \leq 2m$.

14.336 $$\int \frac{x^{p-1}\,dx}{x^{2m}-a^{2m}} = \frac{1}{2ma^{2m-p}}\sum_{k=1}^{m-1}\cos\frac{kp\pi}{m}\ln\left(x^2-2ax\cos\frac{k\pi}{m}+a^2\right) - \frac{1}{ma^{2m-p}}\sum_{k=1}^{m-1}\sin\frac{kp\pi}{m}\tan^{-1}\left(\frac{x-a\cos(k\pi/m)}{a\sin(k\pi/m)}\right) + \frac{1}{2ma^{2m-p}}\{\ln(x-a)+(-1)^p\ln(x+a)\}$$

where $0 < p \leq 2m$.

14.337 $$\int \frac{x^{p-1}\,dx}{x^{2m+1}+a^{2m+1}} = \frac{2(-1)^{p-1}}{(2m+1)a^{2m-p+1}}\sum_{k=1}^{m}\sin\frac{2kp\pi}{2m+1}\tan^{-1}\left(\frac{x+a\cos[2k\pi/(2m+1)]}{a\sin[2k\pi/(2m+1)]}\right) - \frac{(-1)^{p-1}}{(2m+1)a^{2m-p+1}}\sum_{k=1}^{m}\cos\frac{2kp\pi}{2m+1}\ln\left(x^2+2ax\cos\frac{2k\pi}{2m+1}+a^2\right) + \frac{(-1)^{p-1}\ln(x+a)}{(2m+1)a^{2m-p+1}}$$

where $0 < p \leq 2m+1$.

14.338 $$\int \frac{x^{p-1}\,dx}{x^{2m+1}-a^{2m+1}} = \frac{-2}{(2m+1)a^{2m-p+1}}\sum_{k=1}^{m}\sin\frac{2kp\pi}{2m+1}\tan^{-1}\left(\frac{x-a\cos[2k\pi/(2m+1)]}{a\sin[2k\pi/(2m+1)]}\right) + \frac{1}{(2m+1)a^{2m-p+1}}\sum_{k=1}^{m}\cos\frac{2kp\pi}{2m+1}\ln\left(x^2-2ax\cos\frac{2k\pi}{2m+1}+a^2\right) + \frac{\ln(x-a)}{(2m+1)a^{2m-p+1}}$$

where $0 < p \leq 2m+1$.

INTEGRALS INVOLVING $\sin ax$

14.339 $$\int \sin ax\,dx = -\frac{\cos ax}{a}$$

14.340 $$\int x\sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a}$$

14.341 $$\int x^2\sin ax\,dx = \frac{2x}{a^2}\sin ax + \left(\frac{2}{a^3}-\frac{x^2}{a}\right)\cos ax$$

14.342 $$\int x^3\sin ax\,dx = \left(\frac{3x^2}{a^2}-\frac{6}{a^4}\right)\sin ax + \left(\frac{6x}{a^3}-\frac{x^3}{a}\right)\cos ax$$

14.343 $$\int \frac{\sin ax}{x}\,dx = ax - \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} - \cdots$$

14.344 $$\int \frac{\sin ax}{x^2}\,dx = -\frac{\sin ax}{x} + a\int\frac{\cos ax}{x}\,dx$$ [see 14.373]

14.345 $$\int \frac{dx}{\sin ax} = \frac{1}{a}\ln(\csc ax - \cot ax) = \frac{1}{a}\ln\tan\frac{ax}{2}$$

14.346 $$\int \frac{x\,dx}{\sin ax} = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

14.347 $$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

14.348 $$\int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

14.349 $$\int \sin^3 ax\,dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$$

14.350 $$\int \sin^4 ax\,dx = \frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$$

14.351 $$\int \frac{dx}{\sin^2 ax} = -\frac{1}{a}\cot ax$$

14.352 $$\int \frac{dx}{\sin^3 ax} = -\frac{\cos ax}{2a\sin^2 ax} + \frac{1}{2a}\ln\tan\frac{ax}{2}$$

14.353 $$\int \sin px\sin qx\,dx = \frac{\sin(p-q)x}{2(p-q)} - \frac{\sin(p+q)x}{2(p+q)}$$ [If $p = \pm q$, see 14.368.]

14.354 $$\int \frac{dx}{1-\sin ax} = \frac{1}{a}\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right)$$

14.355 $$\int \frac{x\,dx}{1-\sin ax} = \frac{x}{a}\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right) + \frac{2}{a^2}\ln\sin\left(\frac{\pi}{4}-\frac{ax}{2}\right)$$

14.356 $$\int \frac{dx}{1+\sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right)$$

14.357 $$\int \frac{x\,dx}{1+\sin ax} = -\frac{x}{a}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right) + \frac{2}{a^2}\ln\sin\left(\frac{\pi}{4}+\frac{ax}{2}\right)$$

14.358 $$\int \frac{dx}{(1-\sin ax)^2} = \frac{1}{2a}\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right) + \frac{1}{6a}\tan^3\left(\frac{\pi}{4}+\frac{ax}{2}\right)$$

14.359 $$\int \frac{dx}{(1+\sin ax)^2} = -\frac{1}{2a}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right) - \frac{1}{6a}\tan^3\left(\frac{\pi}{4}-\frac{ax}{2}\right)$$

14.360 $$\int \frac{dx}{p+q\sin ax} = \begin{cases} \dfrac{2}{a\sqrt{p^2-q^2}}\tan^{-1}\dfrac{p\tan\frac{1}{2}ax+q}{\sqrt{p^2-q^2}} \\ \dfrac{1}{a\sqrt{q^2-p^2}}\ln\left(\dfrac{p\tan\frac{1}{2}ax+q-\sqrt{q^2-p^2}}{p\tan\frac{1}{2}ax+q+\sqrt{q^2-p^2}}\right) \end{cases}$$

If $p=\pm q$ see 14.354 and 14.356.

14.361 $$\int \frac{dx}{(p+q\sin ax)^2} = \frac{q\cos ax}{a(p^2-q^2)(p+q\sin ax)} + \frac{p}{p^2-q^2}\int \frac{dx}{p+q\sin ax}$$

If $p=\pm q$ see 14.358 and 14.359.

14.362 $$\int \frac{dx}{p^2+q^2\sin^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}}\tan^{-1}\frac{\sqrt{p^2+q^2}\tan ax}{p}$$

14.363 $$\int \frac{dx}{p^2-q^2\sin^2 ax} = \begin{cases} \dfrac{1}{ap\sqrt{p^2-q^2}}\tan^{-1}\dfrac{\sqrt{p^2-q^2}\tan ax}{p} \\ \dfrac{1}{2ap\sqrt{q^2-p^2}}\ln\left(\dfrac{\sqrt{q^2-p^2}\tan ax+p}{\sqrt{q^2-p^2}\tan ax-p}\right) \end{cases}$$

14.364 $$\int x^m\sin ax\,dx = -\frac{x^m\cos ax}{a} + \frac{mx^{m-1}\sin ax}{a^2} - \frac{m(m-1)}{a^2}\int x^{m-2}\sin ax\,dx$$

14.365 $$\int \frac{\sin ax}{x^n}dx = -\frac{\sin ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}}dx \qquad \text{[see 14.395]}$$

14.366 $$\int \sin^n ax\,dx = -\frac{\sin^{n-1}ax\cos ax}{an} + \frac{n-1}{n}\int \sin^{n-2}ax\,dx$$

14.367 $$\int \frac{dx}{\sin^n ax} = \frac{-\cos ax}{a(n-1)\sin^{n-1}ax} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2}ax}$$

14.368 $$\int \frac{x\,dx}{\sin^n ax} = \frac{-x\cos ax}{a(n-1)\sin^{n-1}ax} - \frac{1}{a^2(n-1)(n-2)\sin^{n-2}ax} + \frac{n-2}{n-1}\int \frac{x\,dx}{\sin^{n-2}ax}$$

INTEGRALS INVOLVING cos ax

14.369 $$\int \cos ax\,dx = \frac{\sin ax}{a}$$

14.370 $$\int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a}$$

14.371 $$\int x^2\cos ax\,dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin ax$$

14.372 $$\int x^3\cos ax\,dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\sin ax$$

14.373 $$\int \frac{\cos ax}{x}dx = \ln x - \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} - \frac{(ax)^6}{6\cdot 6!} + \cdots$$

14.374 $$\int \frac{\cos ax}{x^2}dx = -\frac{\cos ax}{x} - a\int \frac{\sin ax}{x}dx \qquad \text{[See 14.343]}$$

14.375 $$\int \frac{dx}{\cos ax} = \frac{1}{a}\ln(\sec ax+\tan ax) = \frac{1}{a}\ln\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right)$$

14.376 $$\int \frac{x\,dx}{\cos ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n(ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

14.377 $$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

14.378 $$\int x\cos^2 ax\,dx = \frac{x^2}{4} + \frac{x\sin 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

14.379 $$\int \cos^3 ax\,dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a}$$

14.380 $$\int \cos^4 ax\,dx = \frac{3x}{8} + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$$

14.381 $$\int \frac{dx}{\cos^2 ax} = \frac{\tan ax}{a}$$

14.382 $$\int \frac{dx}{\cos^3 ax} = \frac{\sin ax}{2a\cos^2 ax} + \frac{1}{2a}\ln\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right)$$

14.383 $$\int \cos ax\cos px\,dx = \frac{\sin(a-p)x}{2(a-p)} + \frac{\sin(a+p)x}{2(a+p)} \qquad \text{[If } a=\pm p\text{, see 14.377.]}$$

14.384 $$\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2}$$

14.385 $$\int \frac{x\,dx}{1-\cos ax} = -\frac{x}{a}\cot\frac{ax}{2} + \frac{2}{a^2}\ln\sin\frac{ax}{2}$$

14.386 $$\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2}$$

14.387 $$\int \frac{x\,dx}{1+\cos ax} = \frac{x}{a}\tan\frac{ax}{2} + \frac{2}{a^2}\ln\cos\frac{ax}{2}$$

14.388 $$\int \frac{dx}{(1-\cos ax)^2} = -\frac{1}{2a}\cot\frac{ax}{2} - \frac{1}{6a}\cot^3\frac{ax}{2}$$

14.389 $$\int \frac{dx}{(1+\cos ax)^2} = \frac{1}{2a}\tan\frac{ax}{2} + \frac{1}{6a}\tan^3\frac{ax}{2}$$

INTEGRALS INVOLVING sin ax AND cos ax

14.399 $$\int \sin ax\cos ax\,dx = \frac{\sin^2 ax}{2a}$$

14.400 $$\int \sin px\cos qx\,dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}$$

14.401 $$\int \sin^n ax\cos ax\,dx = \frac{\sin^{n+1}ax}{(n+1)a} \qquad \text{[If } n=-1\text{, see 14.440.]}$$

14.402 $$\int \cos^n ax\sin ax\,dx = -\frac{\cos^{n+1}ax}{(n+1)a} \qquad \text{[If } n=-1\text{, see 14.429.]}$$

14.403 $$\int \sin^2 ax\cos^2 ax\,dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

14.404 $$\int \frac{dx}{\sin ax\cos ax} = \frac{1}{a}\ln\tan ax$$

14.405 $$\int \frac{dx}{\sin^2 ax\cos ax} = \frac{1}{a}\ln\tan\left(\frac{\pi}{4}+\frac{ax}{2}\right) - \frac{1}{a\sin ax}$$

14.406 $$\int \frac{dx}{\sin ax\cos^2 ax} = \frac{1}{a}\ln\tan\frac{ax}{2} + \frac{1}{a\cos ax}$$

14.407 $$\int \frac{dx}{\sin^2 ax\cos^2 ax} = -\frac{2\cot 2ax}{a}$$

14.408 $$\int \frac{\sin^2 ax}{\cos ax}dx = -\frac{\sin ax}{a} + \frac{1}{a}\ln\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)$$

14.409 $$\int \frac{\cos^2 ax}{\sin ax}dx = \frac{\cos ax}{a} + \frac{1}{a}\ln\tan\frac{ax}{2}$$

14.410 $$\int \frac{dx}{\cos ax(1\pm\sin ax)} = \mp\frac{1}{2a(1\pm\sin ax)} + \frac{1}{2a}\ln\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)$$

14.411 $$\int \frac{dx}{\sin ax(1\pm\cos ax)} = \pm\frac{1}{2a(1\pm\cos ax)} + \frac{1}{2a}\ln\tan\frac{ax}{2}$$

14.412 $$\int \frac{dx}{\sin ax\pm\cos ax} = \frac{1}{a\sqrt{2}}\ln\tan\left(\frac{ax}{2}\pm\frac{\pi}{8}\right)$$

14.413 $$\int \frac{\sin ax\,dx}{\sin ax\pm\cos ax} = \frac{x}{2} \mp \frac{1}{2a}\ln(\sin ax\pm\cos ax)$$

14.414 $$\int \frac{\cos ax\,dx}{\sin ax\pm\cos ax} = \pm\frac{x}{2} + \frac{1}{2a}\ln(\sin ax\pm\cos ax)$$

14.415 $$\int \frac{\sin ax\,dx}{p+q\cos ax} = -\frac{1}{aq}\ln(p+q\cos ax)$$

14.416 $$\int \frac{\cos ax\,dx}{p+q\sin ax} = \frac{1}{aq}\ln(p+q\sin ax)$$

14.417 $$\int \frac{\sin ax\,dx}{(p+q\cos ax)^n} = \frac{1}{aq(n-1)(p+q\cos ax)^{n-1}}$$

14.418 $$\int \frac{\cos ax\,dx}{(p+q\sin ax)^n} = \frac{1}{aq(n-1)(p+q\sin ax)^{n-1}}$$

14.419 $$\int \frac{dx}{p\sin ax+q\cos ax} = \frac{1}{a\sqrt{p^2+q^2}}\ln\tan\left(\frac{ax+\tan^{-1}(q/p)}{2}\right)$$

14.420 $$\int \frac{dx}{p\sin ax+q\cos ax+r} = \begin{cases} \dfrac{2}{a\sqrt{r^2-p^2-q^2}}\tan^{-1}\left(\dfrac{p+(r-q)\tan(ax/2)}{\sqrt{r^2-p^2-q^2}}\right) \\ \dfrac{1}{a\sqrt{p^2+q^2-r^2}}\ln\left(\dfrac{p-\sqrt{p^2+q^2-r^2}+(r-q)\tan(ax/2)}{p+\sqrt{p^2+q^2-r^2}+(r-q)\tan(ax/2)}\right) \end{cases}$$

If $r=q$ see 14.421. If $r^2=p^2+q^2$ see 14.422.

14.421 $$\int \frac{dx}{p\sin ax+q(1+\cos ax)} = \frac{1}{ap}\ln\left(q+p\tan\frac{ax}{2}\right)$$

14.422 $$\int \frac{dx}{p\sin ax+q\cos ax\pm\sqrt{p^2+q^2}} = \frac{1}{a\sqrt{p^2+q^2}}\tan\left(\frac{\pi}{4}\mp\frac{ax+\tan^{-1}(q/p)}{2}\right)$$

14.423 $$\int \frac{dx}{p^2\sin^2 ax+q^2\cos^2 ax} = \frac{1}{apq}\tan^{-1}\left(\frac{p\tan ax}{q}\right)$$

14.424 $$\int \frac{dx}{p^2\sin^2 ax-q^2\cos^2 ax} = \frac{1}{2apq}\ln\left(\frac{p\tan ax-q}{p\tan ax+q}\right)$$

14.425 $$\int \sin^m ax\cos^n ax\,dx = \begin{cases} -\dfrac{\sin^{m-1}ax\cos^{n+1}ax}{a(m+n)} + \dfrac{m-1}{m+n}\int \sin^{m-2}ax\cos^n ax\,dx \\ \dfrac{\sin^{m+1}ax\cos^{n-1}ax}{a(m+n)} + \dfrac{n-1}{m+n}\int \sin^m ax\cos^{n-2}ax\,dx \end{cases}$$

14.426 $$\int \frac{\sin^m ax}{\cos^n ax}dx = \begin{cases} \dfrac{\sin^{m-1}ax}{a(n-1)\cos^{n-1}ax} - \dfrac{m-1}{n-1}\int \dfrac{\sin^{m-2}ax}{\cos^{n-2}ax}dx \\ \dfrac{\sin^{m+1}ax}{a(n-1)\cos^{n-1}ax} - \dfrac{m-n+2}{n-1}\int \dfrac{\sin^m ax}{\cos^{n-2}ax}dx \\ \dfrac{-\sin^{m-1}ax}{a(m-n)\cos^{n-1}ax} + \dfrac{m-1}{m-n}\int \dfrac{\sin^{m-2}ax}{\cos^n ax}dx \end{cases}$$

14.427 $$\int \frac{\cos^m ax}{\sin^n ax}dx = \begin{cases} \dfrac{-\cos^{m-1}ax}{a(n-1)\sin^{n-1}ax} - \dfrac{m-1}{n-1}\int \dfrac{\cos^{m-2}ax}{\sin^{n-2}ax}dx \\ \dfrac{-\cos^{m+1}ax}{a(n-1)\sin^{n-1}ax} - \dfrac{m-n+2}{n-1}\int \dfrac{\cos^m ax}{\sin^{n-2}ax}dx \\ \dfrac{\cos^{m-1}ax}{a(m-n)\sin^{n-1}ax} + \dfrac{m-1}{m-n}\int \dfrac{\cos^{m-2}ax}{\sin^n ax}dx \end{cases}$$

14.428 $$\int \frac{dx}{\sin^m ax\cos^n ax} = \begin{cases} \dfrac{1}{a(n-1)\sin^{m-1}ax\cos^{n-1}ax} + \dfrac{m+n-2}{n-1}\int \dfrac{dx}{\sin^m ax\cos^{n-2}ax} \\ \dfrac{-1}{a(m-1)\sin^{m-1}ax\cos^{n-1}ax} + \dfrac{m+n-2}{m-1}\int \dfrac{dx}{\sin^{m-2}ax\cos^n ax} \end{cases}$$

INTEGRALS INVOLVING $\tan ax$

14.429 $\int \tan ax\,dx = -\frac{1}{a}\ln\cos ax = \frac{1}{a}\ln\sec ax$

14.430 $\int \tan^2 ax\,dx = \frac{\tan ax}{a} - x$

14.431 $\int \tan^3 ax\,dx = \frac{\tan^2 ax}{2a} + \frac{1}{a}\ln\cos ax$

14.432 $\int \tan^n ax \sec^2 ax\,dx = \frac{\tan^{n+1} ax}{(n+1)a}$

14.433 $\int \frac{\sec^2 ax}{\tan ax}\,dx = \frac{1}{a}\ln\tan ax$

14.434 $\int \frac{dx}{\tan ax} = \frac{1}{a}\ln\sin ax$

14.435 $\int x\tan ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} + \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

14.436 $\int \frac{\tan ax}{x}\,dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

14.437 $\int x\tan^2 ax\,dx = \frac{x\tan ax}{a} + \frac{1}{a^2}\ln\cos ax - \frac{x^2}{2}$

14.438 $\int \frac{dx}{p+q\tan ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)}\ln(q\sin ax + p\cos ax)$

14.439 $\int \tan^n ax\,dx = \frac{\tan^{n-1} ax}{(n-1)a} - \int \tan^{n-2} ax\,dx$

INTEGRALS INVOLVING $\cot ax$

14.440 $\int \cot ax\,dx = \frac{1}{a}\ln\sin ax$

14.441 $\int \cot^2 ax\,dx = -\frac{\cot ax}{a} - x$

14.442 $\int \cot^3 ax\,dx = -\frac{\cot^2 ax}{2a} - \frac{1}{a}\ln\sin ax$

14.443 $\int \cot^n ax \csc^2 ax\,dx = -\frac{\cot^{n+1} ax}{(n+1)a}$

14.444 $\int \frac{\csc^2 ax}{\cot ax}\,dx = -\frac{1}{a}\ln\cot ax$

14.445 $\int \frac{dx}{\cot ax} = -\frac{1}{a}\ln\cos ax$

14.446 $\int x\cot ax\,dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \cdots - \frac{2^{2n}B_n(ax)^{2n+1}}{(2n+1)!} - \cdots\right\}$

14.447 $\int \frac{\cot ax}{x}\,dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \cdots - \frac{2^{2n}B_n(ax)^{2n-1}}{(2n-1)(2n)!} - \cdots$

14.448 $\int x\cot^2 ax\,dx = -\frac{x\cot ax}{a} + \frac{1}{a^2}\ln\sin ax - \frac{x^2}{2}$

14.449 $\int \frac{dx}{p+q\cot ax} = \frac{px}{p^2+q^2} - \frac{q}{a(p^2+q^2)}\ln(p\sin ax + q\cos ax)$

14.450 $\int \cot^n ax\,dx = -\frac{\cot^{n-1} ax}{(n-1)a} - \int \cot^{n-2} ax\,dx$

INTEGRALS INVOLVING $\sec ax$

14.451 $\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax + \tan ax) = \frac{1}{a}\ln\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)$

14.452 $\int \sec^2 ax\,dx = \frac{\tan ax}{a}$

14.453 $\int \sec^3 ax\,dx = \frac{\sec ax\tan ax}{2a} + \frac{1}{2a}\ln(\sec ax + \tan ax)$

14.454 $\int \sec^n ax\tan ax\,dx = \frac{\sec^n ax}{na}$

14.455 $\int \frac{dx}{\sec ax} = \frac{\sin ax}{a}$

14.456 $\int x\sec ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n(ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$

14.457 $\int \frac{\sec ax}{x}\,dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \cdots + \frac{E_n(ax)^{2n}}{2n(2n)!} + \cdots$

14.458 $\int x\sec^2 ax\,dx = \frac{x}{a}\tan ax + \frac{1}{a^2}\ln\cos ax$

14.459 $\int \frac{dx}{q+p\sec ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p+q\cos ax}$

14.460 $\int \sec^n ax\,dx = \frac{\sec^{n-2} ax\tan ax}{a(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} ax\,dx$

INTEGRALS INVOLVING $\csc ax$

14.461 $\int \csc ax\,dx = \frac{1}{a}\ln(\csc ax - \cot ax) = \frac{1}{a}\ln\tan\frac{ax}{2}$

14.462 $\int \csc^2 ax\,dx = -\frac{\cot ax}{a}$

14.463 $\int \csc^3 ax\,dx = -\frac{\csc ax\cot ax}{2a} + \frac{1}{2a}\ln\tan\frac{ax}{2}$

14.464 $\int \csc^n ax\cot ax\,dx = -\frac{\csc^n ax}{na}$

14.465 $\int \frac{dx}{\csc ax} = -\frac{\cos ax}{a}$

14.466 $\int x\csc ax\,dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

14.467 $\int \frac{\csc ax}{x}\,dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

14.468 $\int x\csc^2 ax\,dx = -\frac{x\cot ax}{a} + \frac{1}{a^2}\ln\sin ax$

14.469 $\int \frac{dx}{q+p\csc ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p+q\sin ax}$ [See 14.360]

14.470 $\int \csc^n ax\,dx = -\frac{\csc^{n-2} ax\cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} ax\,dx$

INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

14.471 $\int \sin^{-1}\frac{x}{a}\,dx = x\sin^{-1}\frac{x}{a} + \sqrt{a^2-x^2}$

14.472 $\int x\sin^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\sin^{-1}\frac{x}{a} + \frac{x\sqrt{a^2-x^2}}{4}$

14.473 $\int x^2\sin^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\sin^{-1}\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

14.474 $\int \frac{\sin^{-1}(x/a)}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{2\cdot3\cdot3} + \frac{1\cdot3(x/a)^5}{2\cdot4\cdot5\cdot5} + \frac{1\cdot3\cdot5(x/a)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots$

14.475 $\int \frac{\sin^{-1}(x/a)}{x^2}\,dx = -\frac{\sin^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

14.476 $\int\left(\sin^{-1}\frac{x}{a}\right)^2 dx = x\left(\sin^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\sin^{-1}\frac{x}{a}$

14.477 $\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$

14.478 $\int x\cos^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{a^2-x^2}}{4}$

14.479 $\int x^2\cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

14.480 $\int \frac{\cos^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\sin^{-1}(x/a)}{x}\,dx$ [See 14.474]

14.481 $\int \frac{\cos^{-1}(x/a)}{x^2}\,dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

14.482 $\int\left(\cos^{-1}\frac{x}{a}\right)^2 dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\cos^{-1}\frac{x}{a}$

14.483 $\int \tan^{-1}\frac{x}{a}\,dx = x\tan^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$

14.484 $\int x\tan^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\tan^{-1}\frac{x}{a} - \frac{ax}{2}$

14.485 $\int x^2\tan^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\tan^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$

14.486 $\int \frac{\tan^{-1}(x/a)}{x}\,dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \cdots$

14.487 $\int \frac{\tan^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\tan^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

14.488 $\int \cot^{-1}\frac{x}{a}\,dx = x\cot^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$

14.489 $\int x\cot^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\cot^{-1}\frac{x}{a} + \frac{ax}{2}$

14.490 $\int x^2\cot^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cot^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$

14.491 $\int \frac{\cot^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\tan^{-1}(x/a)}{x}\,dx$ [See 14.486]

14.492 $\int \frac{\cot^{-1}(x/a)}{x^2}\,dx = -\frac{\cot^{-1}(x/a)}{x} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

14.493 $\int \sec^{-1}\frac{x}{a}\,dx = \begin{cases} x\sec^{-1}\frac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ x\sec^{-1}\frac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right) & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$

14.494 $\int x\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\sec^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2} & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2}\sec^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2} & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$

14.495 $\int x^2\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\sec^{-1}\frac{x}{a} - \frac{ax\sqrt{x^2-a^2}}{6} - \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3}\sec^{-1}\frac{x}{a} + \frac{ax\sqrt{x^2-a^2}}{6} + \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right) & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$

14.496 $\int \frac{\sec^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x + \frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(a/x)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$

14.497 $\int \frac{\sec^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\sec^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{ax} & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ -\frac{\sec^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{ax} & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$

14.498 $\int \csc^{-1}\frac{x}{a}\,dx = \begin{cases} x\csc^{-1}\frac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \csc^{-1}\frac{x}{a} < \frac{\pi}{2} \\ x\csc^{-1}\frac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right) & -\frac{\pi}{2} < \csc^{-1}\frac{x}{a} < 0 \end{cases}$

14.499 $\int x\csc^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\csc^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2} & 0 < \csc^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2}\csc^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2} & -\frac{\pi}{2} < \csc^{-1}\frac{x}{a} < 0 \end{cases}$

14.500 $\int x^2\csc^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\csc^{-1}\frac{x}{a} + \frac{ax\sqrt{x^2-a^2}}{6} + \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right) & 0 < \csc^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3}\csc^{-1}\frac{x}{a} - \frac{ax\sqrt{x^2-a^2}}{6} - \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right) & -\frac{\pi}{2} < \csc^{-1}\frac{x}{a} < 0 \end{cases}$

14.501 $\int \frac{\csc^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(a/x)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots\right)$

14.502 $\int \frac{\csc^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\csc^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{ax} & 0 < \csc^{-1}\frac{x}{a} < \frac{\pi}{2} \\ -\frac{\csc^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{ax} & -\frac{\pi}{2} < \csc^{-1}\frac{x}{a} < 0 \end{cases}$

14.503 $\int x^m \sin^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\sin^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

14.504 $\int x^m \cos^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cos^{-1}\frac{x}{a} + \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

14.505 $\int x^m \tan^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\tan^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$

14.506 $\int x^m \cot^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cot^{-1}\frac{x}{a} + \frac{a}{m+1}\int \frac{x^{m+1}}{x^2+a^2}\,dx$

14.507 $\int x^m \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}\sec^{-1}(x/a)}{m+1} - \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}\sec^{-1}(x/a)}{m+1} + \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$

14.508 $\int x^m \csc^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}\csc^{-1}(x/a)}{m+1} + \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \csc^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}\csc^{-1}(x/a)}{m+1} - \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2-a^2}} & -\frac{\pi}{2} < \csc^{-1}\frac{x}{a} < 0 \end{cases}$

INTEGRALS INVOLVING e^{ax}

14.509 $\int e^{ax}\,dx = \frac{e^{ax}}{a}$

14.510 $\int xe^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$

14.511 $\int x^2e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$

14.512 $\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

$= \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right)$

14.513 $\int \frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot 1!} + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots$

14.514 $\int \frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{e^{ax}}{x^{n-1}}\,dx$

14.515 $\int \frac{dx}{p+qe^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+qe^{ax})$

14.516 $\int \frac{dx}{(p+qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+qe^{ax})} - \frac{1}{ap^2}\ln(p+qe^{ax})$

14.517 $\int \frac{dx}{pe^{ax}+qe^{-ax}} = \begin{cases} \frac{1}{a\sqrt{pq}}\tan^{-1}\left(\sqrt{\frac{p}{q}}e^{ax}\right) \\ \frac{1}{2a\sqrt{-pq}}\ln\left(\frac{e^{ax}-\sqrt{-q/p}}{e^{ax}+\sqrt{-q/p}}\right) \end{cases}$

14.518 $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$

14.519 $\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2}$

14.520 $\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\sin bx - 2ab\cos bx\}}{(a^2+b^2)^2}$

14.521 $\int xe^{ax}\cos bx\,dx = \frac{xe^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\cos bx + 2ab\sin bx\}}{(a^2+b^2)^2}$

14.522 $\int e^{ax}\ln x\,dx = \frac{e^{ax}\ln x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x}\,dx$

14.523 $\int e^{ax}\sin^n bx\,dx = \frac{e^{ax}\sin^{n-1}bx}{a^2+n^2b^2}(a\sin bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\sin^{n-2}bx\,dx$

14.524 $\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1}bx}{a^2+n^2b^2}(a\cos bx + nb\sin bx) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\cos^{n-2}bx\,dx$

INTEGRALS INVOLVING $\ln x$

14.525 $\int \ln x\,dx = x\ln x - x$

14.526 $\int x\ln x\,dx = \frac{x^2}{2}(\ln x - \frac{1}{2})$

14.527 $\int x^m\ln x\,dx = \frac{x^{m+1}}{m+1}\left(\ln x - \frac{1}{m+1}\right)$ [If $m=-1$ see 14.528.]

14.528 $\int \frac{\ln x}{x}\,dx = \frac{1}{2}\ln^2 x$

14.529 $\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x}$

14.530 $\int \ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$

14.531 $\int \frac{\ln^n x\,dx}{x} = \frac{\ln^{n+1}x}{n+1}$ [If $n=-1$ see 14.532.]

14.532 $\int \frac{dx}{x\ln x} = \ln(\ln x)$

14.533 $\int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2\cdot 2!} + \frac{\ln^3 x}{3\cdot 3!} + \cdots$

14.534 $\int \frac{x^m\,dx}{\ln x} = \ln(\ln x) + (m+1)\ln x + \frac{(m+1)^2\ln^2 x}{2\cdot 2!} + \frac{(m+1)^3\ln^3 x}{3\cdot 3!} + \cdots$

14.535 $\int \ln^n x\,dx = x\ln^n x - n\int \ln^{n-1}x\,dx$

14.536 $\int x^m\ln^n x\,dx = \frac{x^{m+1}\ln^n x}{m+1} - \frac{n}{m+1}\int x^m\ln^{n-1}x\,dx$

If $m=-1$ see 14.531.

14.537 $\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\tan^{-1}\frac{x}{a}$

14.538 $\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) - 2x + a\ln\left(\frac{x+a}{x-a}\right)$

14.539 $\int x^m\ln(x^2\pm a^2)\,dx = \frac{x^{m+1}\ln(x^2\pm a^2)}{m+1} - \frac{2}{m+1}\int \frac{x^{m+2}}{x^2\pm a^2}\,dx$

INTEGRALS INVOLVING $\sinh ax$

14.540 $\int \sinh ax\,dx = \frac{\cosh ax}{a}$

14.541 $\int x\sinh ax\,dx = \frac{x\cosh ax}{a} - \frac{\sinh ax}{a^2}$

14.542 $\int x^2\sinh ax\,dx = \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\cosh ax - \frac{2x}{a^2}\sinh ax$

14.543 $\int \frac{\sinh ax}{x}\,dx = ax + \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} + \cdots$

14.544 $\int \frac{\sinh ax}{x^2}\,dx = -\frac{\sinh ax}{x} + a\int \frac{\cosh ax}{x}\,dx$ [See 14.565]

14.545 $\int \frac{dx}{\sinh ax} = \frac{1}{a}\ln\tanh\frac{ax}{2}$

if n = positive integer

14.546 $\int \frac{x\,dx}{\sinh ax} = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \cdots + \frac{2(-1)^n(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

14.547 $\int \sinh^2 ax\,dx = \frac{\sinh ax\cosh ax}{2a} - \frac{x}{2}$

14.548 $\int x\sinh^2 ax\,dx = \frac{x\sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2} - \frac{x^2}{4}$

14.549 $\int \frac{dx}{\sinh^2 ax} = -\frac{\coth ax}{a}$

14.550 $\int \sinh ax\sinh px\,dx = \frac{\sinh(a+p)x}{2(a+p)} - \frac{\sinh(a-p)x}{2(a-p)}$

For $a = \pm p$ see 14.547.

14.551 $\int \sinh ax\sin px\,dx = \frac{a\cosh ax\sin px - p\sinh ax\cos px}{a^2+p^2}$

14.552 $\int \sinh ax\cos px\,dx = \frac{a\cosh ax\cos px + p\sinh ax\sin px}{a^2+p^2}$

14.553 $\int \frac{dx}{p+q\sinh ax} = \frac{1}{a\sqrt{p^2+q^2}}\ln\left(\frac{qe^{ax}+p-\sqrt{p^2+q^2}}{qe^{ax}+p+\sqrt{p^2+q^2}}\right)$

14.554 $\int \frac{dx}{(p+q\sinh ax)^2} = \frac{-q\cosh ax}{a(p^2+q^2)(p+q\sinh ax)} + \frac{p}{p^2+q^2}\int \frac{dx}{p+q\sinh ax}$

14.555 $\int \frac{dx}{p^2+q^2\sinh^2 ax} = \begin{cases} \frac{1}{ap\sqrt{q^2-p^2}}\tan^{-1}\frac{\sqrt{q^2-p^2}\tanh ax}{p} \\ \frac{1}{2ap\sqrt{p^2-q^2}}\ln\left(\frac{p+\sqrt{p^2-q^2}\tanh ax}{p-\sqrt{p^2-q^2}\tanh ax}\right) \end{cases}$

14.556 $\int \frac{dx}{p^2-q^2\sinh^2 ax} = \frac{1}{2ap\sqrt{p^2+q^2}}\ln\left(\frac{p+\sqrt{p^2+q^2}\tanh ax}{p-\sqrt{p^2+q^2}\tanh ax}\right)$

14.557 $\int x^m\sinh ax\,dx = \frac{x^m\cosh ax}{a} - \frac{m}{a}\int x^{m-1}\cosh ax\,dx$ [See 14.585]

14.558 $\int \sinh^n ax\,dx = \frac{\sinh^{n-1} ax\cosh ax}{an} - \frac{n-1}{n}\int \sinh^{n-2} ax\,dx$

14.559 $\int \frac{\sinh ax}{x^n}dx = \frac{-\sinh ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cosh ax}{x^{n-1}}dx$ [See 14.587]

14.560 $\int \frac{dx}{\sinh^n ax} = \frac{-\cosh ax}{a(n-1)\sinh^{n-1} ax} - \frac{n-2}{n-1}\int \frac{dx}{\sinh^{n-2} ax}$

14.561 $\int \frac{x\,dx}{\sinh^n ax} = \frac{-x\cosh ax}{a(n-1)\sinh^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\sinh^{n-2} ax} - \frac{n-2}{n-1}\int \frac{x\,dx}{\sinh^{n-2} ax}$

INTEGRALS INVOLVING cosh ax

14.562 $\int \cosh ax\,dx = \frac{\sinh ax}{a}$

14.563 $\int x\cosh ax\,dx = \frac{x\sinh ax}{a} - \frac{\cosh ax}{a^2}$

14.564 $\int x^2\cosh ax\,dx = -\frac{2x\cosh ax}{a^2} + \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\sinh ax$

14.565 $\int \frac{\cosh ax}{x}dx = \ln x + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} + \frac{(ax)^6}{6\cdot 6!} + \cdots$

14.566 $\int \frac{\cosh ax}{x^2}dx = -\frac{\cosh ax}{x} + a\int \frac{\sinh ax}{x}dx$ [See 14.543]

14.567 $\int \frac{dx}{\cosh ax} = \frac{2}{a}\tan^{-1} e^{ax}$

14.568 $\int \frac{x\,dx}{\cosh ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$

14.569 $\int \cosh^2 ax\,dx = \frac{x}{2} + \frac{\sinh ax\cosh ax}{2a}$

14.570 $\int x\cosh^2 ax\,dx = \frac{x^2}{4} + \frac{x\sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$

14.571 $\int \frac{dx}{\cosh^2 ax} = \frac{\tanh ax}{a}$

14.572 $\int \cosh ax\cosh px\,dx = \frac{\sinh(a-p)x}{2(a-p)} + \frac{\sinh(a+p)x}{2(a+p)}$

14.573 $\int \cosh ax\sin px\,dx = \frac{a\sinh ax\sin px - p\cosh ax\cos px}{a^2+p^2}$

14.574 $\int \cosh ax\cos px\,dx = \frac{a\sinh ax\cos px + p\cosh ax\sin px}{a^2+p^2}$

14.575 $\int \frac{dx}{\cosh ax+1} = \frac{1}{a}\tanh\frac{ax}{2}$

14.576 $\int \frac{dx}{\cosh ax-1} = -\frac{1}{a}\coth\frac{ax}{2}$

14.577 $\int \frac{x\,dx}{\cosh ax+1} = \frac{x}{a}\tanh\frac{ax}{2} - \frac{2}{a^2}\ln\cosh\frac{ax}{2}$

14.578 $\int \frac{x\,dx}{\cosh ax-1} = -\frac{x}{a}\coth\frac{ax}{2} + \frac{2}{a^2}\ln\sinh\frac{ax}{2}$

14.579 $\int \frac{dx}{(\cosh ax+1)^2} = \frac{1}{2a}\tanh\frac{ax}{2} - \frac{1}{6a}\tanh^3\frac{ax}{2}$

14.580 $\int \frac{dx}{(\cosh ax-1)^2} = \frac{1}{2a}\coth\frac{ax}{2} - \frac{1}{6a}\coth^3\frac{ax}{2}$

14.581 $\int \frac{dx}{p+q\cosh ax} = \begin{cases} \frac{2}{a\sqrt{q^2-p^2}}\tan^{-1}\frac{qe^{ax}+p}{\sqrt{q^2-p^2}} \\ \frac{1}{a\sqrt{p^2-q^2}}\ln\left(\frac{qe^{ax}+p-\sqrt{p^2-q^2}}{qe^{ax}+p+\sqrt{p^2-q^2}}\right) \end{cases}$

14.582 $\int \frac{dx}{(p+q\cosh ax)^2} = \frac{q\sinh ax}{a(q^2-p^2)(p+q\cosh ax)} - \frac{p}{q^2-p^2}\int \frac{dx}{p+q\cosh ax}$

14.583 $\int \frac{dx}{p^2-q^2\cosh^2 ax} = \begin{cases} \frac{1}{2ap\sqrt{p^2-q^2}}\ln\left(\frac{p\tanh ax+\sqrt{p^2-q^2}}{p\tanh ax-\sqrt{p^2-q^2}}\right) \\ \frac{-1}{ap\sqrt{q^2-p^2}}\tan^{-1}\frac{p\tanh ax}{\sqrt{q^2-p^2}} \end{cases}$

14.584 $\int \frac{dx}{p^2+q^2\cosh^2 ax} = \begin{cases} \frac{1}{2ap\sqrt{p^2+q^2}}\ln\left(\frac{p\tanh ax+\sqrt{p^2+q^2}}{p\tanh ax-\sqrt{p^2+q^2}}\right) \\ \frac{1}{ap\sqrt{p^2+q^2}}\tan^{-1}\frac{p\tanh ax}{\sqrt{p^2+q^2}} \end{cases}$

14.585 $\int x^m\cosh ax\,dx = \frac{x^m\sinh ax}{a} - \frac{m}{a}\int x^{m-1}\sinh ax\,dx$ [See 14.557]

14.586 $\int \cosh^n ax\,dx = \frac{\cosh^{n-1} ax\sinh ax}{an} + \frac{n-1}{n}\int \cosh^{n-2} ax\,dx$

14.587 $\int \frac{\cosh ax}{x^n}dx = \frac{-\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\sinh ax}{x^{n-1}}dx$ [See 14.559]

14.588 $\int \frac{dx}{\cosh^n ax} = \frac{\sinh ax}{a(n\ \ 1)\cosh^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cosh^{n-2} ax}$

14.589 $\int \frac{x\,dx}{\cosh^n ax} = \frac{x\sinh ax}{a(n-1)\cosh^{n-1} ax} + \frac{1}{(n-1)(n-2)a^2\cosh^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\,dx}{\cosh^{n-2} ax}$

INTEGRALS INVOLVING sinh ax AND cosh ax

14.590 $\int \sinh ax\cosh ax\,dx = \frac{\sinh^2 ax}{2a}$

14.591 $\int \sinh px\cosh qx\,dx = \frac{\cosh(p+q)x}{2(p+q)} + \frac{\cosh(p-q)x}{2(p-q)}$

14.592 $\int \sinh^n ax\cosh ax\,dx = \frac{\sinh^{n+1} ax}{(n+1)a}$ [If $n=-1$, see 14.615.]

14.593 $\int \cosh^n ax\sinh ax\,dx = \frac{\cosh^{n+1} ax}{(n+1)a}$ [If $n=-1$, see 14.604.]

14.594 $\int \sinh^2 ax\cosh^2 ax\,dx = \frac{\sinh 4ax}{32a} - \frac{x}{8}$

14.595 $\int \frac{dx}{\sinh ax\cosh ax} = \frac{1}{a}\ln\tanh ax$

14.596 $\int \frac{dx}{\sinh^2 ax\cosh ax} = -\frac{1}{a}\tan^{-1}\sinh ax - \frac{\operatorname{csch} ax}{a}$

14.597 $\int \frac{dx}{\sinh ax\cosh^2 ax} = \frac{\operatorname{sech} ax}{a} + \frac{1}{a}\ln\tanh\frac{ax}{2}$

14.598 $\int \frac{dx}{\sinh^2 ax\cosh^2 ax} = -\frac{2\coth 2ax}{a}$

14.599 $\int \frac{\sinh^2 ax}{\cosh ax}dx = \frac{\sinh ax}{a} - \frac{1}{a}\tan^{-1}\sinh ax$

14.600 $\int \frac{\cosh^2 ax}{\sinh ax}dx = \frac{\cosh ax}{a} + \frac{1}{a}\ln\tanh\frac{ax}{2}$

14.601 $\int \frac{dx}{\cosh ax\,(1+\sinh ax)} = \frac{1}{2a}\ln\left(\frac{1+\sinh ax}{\cosh ax}\right) + \frac{1}{a}\tan^{-1} e^{ax}$

14.602 $\int \frac{dx}{\sinh ax\,(\cosh ax+1)} = \frac{1}{2a}\ln\tanh\frac{ax}{2} + \frac{1}{2a(\cosh ax+1)}$

14.603 $\int \frac{dx}{\sinh ax\,(\cosh ax-1)} = -\frac{1}{2a}\ln\tanh\frac{ax}{2} - \frac{1}{2a(\cosh ax-1)}$

INTEGRALS INVOLVING tanh ax

14.604 $\int \tanh ax\,dx = \frac{1}{a}\ln\cosh ax$

14.605 $\int \tanh^2 ax\,dx = x - \frac{\tanh ax}{a}$

14.606 $\int \tanh^3 ax\,dx = \frac{1}{a}\ln\cosh ax - \frac{\tanh^2 ax}{2a}$

14.607 $\int \tanh^n ax\operatorname{sech}^2 ax\,dx = \frac{\tanh^{n+1} ax}{(n+1)a}$

14.608 $\int \frac{\operatorname{sech}^2 ax}{\tanh ax}dx = \frac{1}{a}\ln\tanh ax$

14.609 $\int \frac{dx}{\tanh ax} = \frac{1}{a}\ln\sinh ax$

14.610 $\int x\tanh ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \cdots \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

14.611 $\int x\tanh^2 ax\,dx = \frac{x^2}{2} - \frac{x\tanh ax}{a} + \frac{1}{a^2}\ln\cosh ax$

14.612 $$\int \frac{\tanh ax}{x}\,dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} - \cdots \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

14.613 $$\int \frac{dx}{p+q\tanh ax} = \frac{px}{p^2-q^2} - \frac{q}{a(p^2-q^2)}\ln(q\sinh ax + p\cosh ax)$$

14.614 $$\int \tanh^n ax\,dx = \frac{-\tanh^{n-1} ax}{a(n-1)} + \int \tanh^{n-2} ax\,dx$$

INTEGRALS INVOLVING coth ax

14.615 $$\int \coth ax\,dx = \frac{1}{a}\ln\sinh ax$$

14.616 $$\int \coth^2 ax\,dx = x - \frac{\coth ax}{a}$$

14.617 $$\int \coth^3 ax\,dx = \frac{1}{a}\ln\sinh ax - \frac{\coth^2 ax}{2a}$$

14.618 $$\int \coth^n ax\,\operatorname{csch}^2 ax\,dx = -\frac{\coth^{n+1} ax}{(n+1)a}$$

14.619 $$\int \frac{\operatorname{csch}^2 ax}{\coth ax}\,dx = -\frac{1}{a}\ln\coth ax$$

14.620 $$\int \frac{dx}{\coth ax} = \frac{1}{a}\ln\cosh ax$$

14.621 $$\int x\coth ax\,dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \cdots \frac{(-1)^{n-1}2^{2n}B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

14.622 $$\int x\coth^2 ax\,dx = \frac{x^2}{2} - \frac{x\coth ax}{a} + \frac{1}{a^2}\ln\sinh ax$$

14.623 $$\int \frac{\coth ax}{x}\,dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \cdots \frac{(-1)^n 2^{2n}B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

14.624 $$\int \frac{dx}{p+q\coth ax} = \frac{px}{p^2-q^2} - \frac{q}{a(p^2-q^2)}\ln(p\sinh ax + q\cosh ax)$$

14.625 $$\int \coth^n ax\,dx = -\frac{\coth^{n-1} ax}{a(n-1)} + \int \coth^{n-2} ax\,dx$$

INTEGRALS INVOLVING sech ax

14.626 $$\int \operatorname{sech} ax\,dx = \frac{2}{a}\tan^{-1} e^{ax}$$

14.627 $$\int \operatorname{sech}^2 ax\,dx = \frac{\tanh ax}{a}$$

14.628 $$\int \operatorname{sech}^3 ax\,dx = \frac{\operatorname{sech} ax\tanh ax}{2a} + \frac{1}{2a}\tan^{-1}\sinh ax$$

14.629 $$\int \operatorname{sech}^n ax\tanh ax\,dx = -\frac{\operatorname{sech}^n ax}{na}$$

14.630 $$\int \frac{dx}{\operatorname{sech} ax} = \frac{\sinh ax}{a}$$

14.631 $$\int x\operatorname{sech} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots \frac{(-1)^n E_n(ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

14.632 $$\int x\operatorname{sech}^2 ax\,dx = \frac{x\tanh ax}{a} - \frac{1}{a^2}\ln\cosh ax$$

14.633 $$\int \frac{\operatorname{sech} ax}{x}\,dx = \ln x - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \cdots \frac{(-1)^n E_n(ax)^{2n}}{2n(2n)!} + \cdots$$

14.634 $$\int \frac{dx}{q+p\operatorname{sech} ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p+q\cosh ax} \qquad \text{[See 14.581]}$$

14.635 $$\int \operatorname{sech}^n ax\,dx = \frac{\operatorname{sech}^{n-2} ax\tanh ax}{a(n-1)} + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2} ax\,dx$$

INTEGRALS INVOLVING csch ax

14.636 $$\int \operatorname{csch} ax\,dx = \frac{1}{a}\ln\tanh\frac{ax}{2}$$

14.637 $$\int \operatorname{csch}^2 ax\,dx = -\frac{\coth ax}{a}$$

14.638 $$\int \operatorname{csch}^3 ax\,dx = -\frac{\operatorname{csch} ax\coth ax}{2a} - \frac{1}{2a}\ln\tanh\frac{ax}{2}$$

14.639 $$\int \operatorname{csch}^n ax\coth ax\,dx = -\frac{\operatorname{csch}^n ax}{na}$$

14.640 $$\int \frac{dx}{\operatorname{csch} ax} = \frac{1}{a}\cosh ax$$

14.641 $$\int x\operatorname{csch} ax\,dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(-1)^n(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

14.642 $$\int x\operatorname{csch}^2 ax\,dx = -\frac{x\coth ax}{a} + \frac{1}{a^2}\ln\sinh ax$$

14.643 $$\int \frac{\operatorname{csch} ax}{x}\,dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots \frac{(-1)^n 2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

14.644 $$\int \frac{dx}{q+p\operatorname{csch} ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p+q\sinh ax} \qquad \text{[See 14.553]}$$

14.645 $$\int \operatorname{csch}^n ax\,dx = \frac{-\operatorname{csch}^{n-2} ax\coth ax}{a(n-1)} - \frac{n-2}{n-1}\int \operatorname{csch}^{n-2} ax\,dx$$

INTEGRALS INVOLVING INVERSE HYPERBOLIC FUNCTIONS

14.646 $$\int \sinh^{-1}\frac{x}{a}\,dx = x\sinh^{-1}\frac{x}{a} - \sqrt{x^2+a^2}$$

14.647 $$\int x\sinh^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}+\frac{a^2}{4}\right)\sinh^{-1}\frac{x}{a} - \frac{x\sqrt{x^2+a^2}}{4}$$

14.648 $$\int x^2\sinh^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\sinh^{-1}\frac{x}{a} + \frac{(2a^2-x^2)\sqrt{x^2+a^2}}{9}$$

14.649 $$\int \frac{\sinh^{-1}(x/a)}{x}\,dx = \begin{cases} \dfrac{x}{a} - \dfrac{(x/a)^3}{2\cdot3\cdot3} + \dfrac{1\cdot3(x/a)^5}{2\cdot4\cdot5\cdot5} - \dfrac{1\cdot3\cdot5(x/a)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots & |x|<a \\ \dfrac{\ln^2(2x/a)}{2} - \dfrac{(a/x)^2}{2\cdot2\cdot2} + \dfrac{1\cdot3(a/x)^4}{2\cdot4\cdot4\cdot4} - \dfrac{1\cdot3\cdot5(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} + \cdots & x>a \\ -\dfrac{\ln^2(-2x/a)}{2} + \dfrac{(a/x)^2}{2\cdot2\cdot2} - \dfrac{1\cdot3(a/x)^4}{2\cdot4\cdot4\cdot4} + \dfrac{1\cdot3\cdot5(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} - \cdots & x<-a \end{cases}$$

14.650 $$\int \frac{\sinh^{-1}(x/a)}{x^2}\,dx = -\frac{\sinh^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

14.651 $$\int \cosh^{-1}\frac{x}{a}\,dx = \begin{cases} x\cosh^{-1}(x/a) - \sqrt{x^2-a^2}, & \cosh^{-1}(x/a) > 0 \\ x\cosh^{-1}(x/a) + \sqrt{x^2-a^2}, & \cosh^{-1}(x/a) < 0 \end{cases}$$

14.652 $$\int x\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{1}{4}(2x^2-a^2)\cosh^{-1}(x/a) - \frac{1}{4}x\sqrt{x^2-a^2}, & \cosh^{-1}(x/a) > 0 \\ \frac{1}{4}(2x^2-a^2)\cosh^{-1}(x/a) + \frac{1}{4}x\sqrt{x^2-a^2}, & \cosh^{-1}(x/a) < 0 \end{cases}$$

14.653 $$\int x^2\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{1}{3}x^3\cosh^{-1}(x/a) - \frac{1}{9}(x^2+2a^2)\sqrt{x^2-a^2}, & \cosh^{-1}(x/a) > 0 \\ \frac{1}{3}x^3\cosh^{-1}(x/a) + \frac{1}{9}(x^2+2a^2)\sqrt{x^2-a^2}, & \cosh^{-1}(x/a) < 0 \end{cases}$$

14.654 $$\int \frac{\cosh^{-1}(x/a)}{x}\,dx = \pm\left[\frac{1}{2}\ln^2(2x/a) + \frac{(a/x)^2}{2\cdot2\cdot2} + \frac{1\cdot3(a/x)^4}{2\cdot4\cdot4\cdot4} + \frac{1\cdot3\cdot5(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} + \cdots\right]$$
$+$ if $\cosh^{-1}(x/a) > 0$, $-$ if $\cosh^{-1}(x/a) < 0$

14.655 $$\int \frac{\cosh^{-1}(x/a)}{x^2}\,dx = -\frac{\cosh^{-1}(x/a)}{x} \mp \frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right) \quad \begin{matrix}[-\text{ if } \cosh^{-1}(x/a)>0,\\ +\text{ if } \cosh^{-1}(x/a)<0]\end{matrix}$$

14.656 $$\int \tanh^{-1}\frac{x}{a}\,dx = x\tanh^{-1}\frac{x}{a} + \frac{a}{2}\ln(a^2-x^2)$$

14.657 $$\int x\tanh^{-1}\frac{x}{a}\,dx = \frac{ax}{2} + \frac{1}{2}(x^2-a^2)\tanh^{-1}\frac{x}{a}$$

14.658 $$\int x^2\tanh^{-1}\frac{x}{a}\,dx = \frac{ax^2}{6} + \frac{x^3}{3}\tanh^{-1}\frac{x}{a} + \frac{a^3}{6}\ln(a^2-x^2)$$

14.659 $$\int \frac{\tanh^{-1}(x/a)}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} + \cdots$$

14.660 $$\int \frac{\tanh^{-1}(x/a)}{x^2}\,dx = -\frac{\tanh^{-1}(x/a)}{x} + \frac{1}{2a}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

14.661 $$\int \coth^{-1}\frac{x}{a}\,dx = x\coth^{-1}x + \frac{a}{2}\ln(x^2-a^2)$$

14.662 $$\int x\coth^{-1}\frac{x}{a}\,dx = \frac{ax}{2} + \frac{1}{2}(x^2-a^2)\coth^{-1}\frac{x}{a}$$

14.663 $$\int x^2\coth^{-1}\frac{x}{a}\,dx = \frac{ax^2}{6} + \frac{x^3}{3}\coth^{-1}\frac{x}{a} + \frac{a^3}{6}\ln(x^2-a^2)$$

14.664 $$\int \frac{\coth^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \cdots\right)$$

14.665 $$\int \frac{\coth^{-1}(x/a)}{x^2}\,dx = -\frac{\coth^{-1}(x/a)}{x} + \frac{1}{2a}\ln\left(\frac{x^2}{x^2-a^2}\right)$$

14.666 $$\int \operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} x\operatorname{sech}^{-1}(x/a) + a\sin^{-1}(x/a), & \operatorname{sech}^{-1}(x/a) > 0 \\ x\operatorname{sech}^{-1}(x/a) - a\sin^{-1}(x/a), & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

14.667 $$\int x\operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{1}{2}x^2\operatorname{sech}^{-1}(x/a) - \frac{1}{2}a\sqrt{a^2-x^2}, & \operatorname{sech}^{-1}(x/a) > 0 \\ \frac{1}{2}x^2\operatorname{sech}^{-1}(x/a) + \frac{1}{2}a\sqrt{a^2-x^2}, & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

14.668 $$\int \frac{\operatorname{sech}^{-1}(x/a)}{x}\,dx = \begin{cases} -\frac{1}{2}\ln(a/x)\ln(4a/x) - \dfrac{(x/a)^2}{2\cdot2\cdot2} - \dfrac{1\cdot3(x/a)^4}{2\cdot4\cdot4\cdot4} - \cdots, & \operatorname{sech}^{-1}(x/a) > 0 \\ \frac{1}{2}\ln(a/x)\ln(4a/x) + \dfrac{(x/a)^2}{2\cdot2\cdot2} + \dfrac{1\cdot3(x/a)^4}{2\cdot4\cdot4\cdot4} + \cdots, & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

14.669 $$\int \operatorname{csch}^{-1}\frac{x}{a}\,dx = x\operatorname{csch}^{-1}\frac{x}{a} \pm a\sinh^{-1}\frac{x}{a} \qquad [+\text{ if } x>0, -\text{ if } x<0]$$

14.670 $$\int x\operatorname{csch}^{-1}\frac{x}{a}\,dx = \frac{x^2}{2}\operatorname{csch}^{-1}\frac{x}{a} \pm \frac{a\sqrt{x^2+a^2}}{2} \qquad [+\text{ if } x>0, -\text{ if } x<0]$$

14.671 $$\int \frac{\operatorname{csch}^{-1}(x/a)}{x}\,dx = \begin{cases} \frac{1}{2}\ln(x/a)\ln(4a/x) + \dfrac{1(x/a)^2}{2\cdot2\cdot2} - \dfrac{1\cdot3(x/a)^4}{2\cdot4\cdot4\cdot4} + \cdots & 0<x<a \\ \frac{1}{2}\ln(-x/a)\ln(-x/4a) - \dfrac{(x/a)^2}{2\cdot2\cdot2} + \dfrac{1\cdot3(x/a)^4}{2\cdot4\cdot4\cdot4} - \cdots & -a<x<0 \\ -\dfrac{a}{x} + \dfrac{(a/x)^3}{2\cdot3\cdot3} - \dfrac{1\cdot3(a/x)^5}{2\cdot4\cdot5\cdot5} + \cdots & |x|>a \end{cases}$$

14.672 $$\int x^m\sinh^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\sinh^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{x^2+a^2}}\,dx$$

14.673 $$\int x^m\cosh^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}}{m+1}\cosh^{-1}\dfrac{x}{a} - \dfrac{1}{m+1}\displaystyle\int \frac{x^{m+1}}{\sqrt{x^2-a^2}}\,dx & \cosh^{-1}(x/a) > 0 \\ \dfrac{x^{m+1}}{m+1}\cosh^{-1}\dfrac{x}{a} + \dfrac{1}{m+1}\displaystyle\int \frac{x^{m+1}}{\sqrt{x^2-a^2}}\,dx & \cosh^{-1}(x/a) < 0 \end{cases}$$

14.674 $$\int x^m\tanh^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\tanh^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{a^2-x^2}\,dx$$

14.675 $$\int x^m\coth^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\coth^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{a^2-x^2}\,dx$$

14.676 $$\int x^m\operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\dfrac{x}{a} + \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{a^2-x^2}} & \operatorname{sech}^{-1}(x/a) > 0 \\ \dfrac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\dfrac{x}{a} - \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{a^2-x^2}} & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

14.677 $$\int x^m\operatorname{csch}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{csch}^{-1}\frac{x}{a} \pm \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2+a^2}} \qquad [+\text{ if } x>0, -\text{ if } x<0]$$